MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 92 No. 5

MEETING OF 1932 MARCH 11

Dr. H. Knox-Shaw, President, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Wilfred Edward Candler, B.A., Trinity College, Cambridge (proposed by W. M. Smart) ;

W. H. Darracott, 911 Jessie Avenue, Winnipeg, Canada (proposed by E. A. Milne) ;

Norman Fairclough, M.Sc., Huyton, 27 Alteryn Close, Newport, Mon. (proposed by E. A. Milne) ;

James Chisholm Jenkinson, J.P., c/o Salvation Army, Uralla, N.S.W., Australia (proposed by W. F. Gale) ;

Geoffrey Francis Kellaway, Coker Hill, West Coker, Yeovil, Somerset (proposed by T. E. R. Phillips) ;

Cecil Harris Leonard, B.Sc., Arwell, Pen-y-groes, N. Wales (proposed by J. R. Owens) ;

Maurice Rupert Meehan, The High School, Marlborough, Wilts (proposed by W. E. Cooper) ; and

Mrs. Natalia S. Samoilowa-Jachontowa, Astronomical Institute, Leningrad, and 19 Karpovka, Ap. 41, Leningrad 22, U.S.S.R. (proposed by A. Belopolsky).

One hundred and forty-two presents were announced as having been received since the last ordinary meeting, including :—

Albany, Dudley Observatory, Albany Catalogue of 20811 Stars for the Epoch 1910 [Carnegie Institution Publication No. 419];

Astrographic Chart : Bordeaux Observatory, zones +12°, +14°, +16° (21 charts) ; Paris Observatory, zones +18°, +20°, +22° (23 charts) ; San Fernando Observatory, zone −5° (10 charts) ; Toulouse Observatory, zones +5°, +7°, +9° (15 charts);
Eddington, Sir A. S., "The Expanding Universe";
Kharkov Astronomical Observatory, Publications, vol. 3;
Sonneberg, Sternwarte, Mitteilungen, Nr. 12–20.

THE "GUILLOTINE FACTOR" IN STELLAR OPACITY.

Sir A. S. Eddington, F.R.S.

1. Calculations of the absorption coefficient by wave-mechanics are found to be in close agreement with Kramers's theory, which was based on a rather strained application of the Correspondence Principle. We may therefore continue to base the calculation of stellar opacity on Kramers's formulæ ; the results will almost equally well represent the most up-to-date physical theory.

When applied to the interior of a star, Kramers's results lead to a coefficient of opacity *

$$k = \kappa\rho/\mu T^{\frac{7}{2}}(1+f)g, \tag{1}$$

where

$$\kappa = 7{\cdot}44\frac{128\pi}{3\sqrt{3}}\frac{Z^2}{A}\frac{e^6R}{\mathrm{H}^2am^2c^4hu_0} \tag{2}$$

and is therefore a known constant for any element ; f is the ratio of the number of ions to the number of free electrons.

The factor g, which I have called the guillotine factor, is nearly unity in typical giant stars ; but it causes a considerable reduction of the calculated opacity for the Sun and smaller stars. Hitherto only rough calculations of g have been given.† Now that we are more convinced of the trustworthiness of Kramers's theory it is desirable to improve the calculation of this factor, and the purpose of the present paper is to obtain more accurate values.

It now seems probable that hydrogen constitutes a considerable proportion of the material of a star, and it is particularly with a view to developing this hypothesis that I have felt the need for more accurate guillotine factors. The hydrogen ions themselves contribute practically nothing to the absorption, and it is to the remaining elements that the formulæ (1) and (2) are applied. For calculating Z, A, μ, we should naturally replace the mixture of elements (excluding the hydrogen) by an equivalent mean element ; but in calculating the opacity it is necessary to distinguish between a mean element and a pure (or greatly preponderant) element. A pure element has con-

* *Internal Constitution of the Stars*, equation (158·1). The notation of the physical constants appearing in (2) and their numerical values will be found in the Appendix, *I.C.S.*, p. 395.

† *I.C.S.*, p. 233, Table XXIX.

siderably lower opacity than a mixture, because it leaves thin places in the spectrum, like windows for the radiation to pour through ; a mixture gives more even strength and is more efficient in blocking radiation.

The formulæ here developed refer to a mixture, not to a pure element. It is a possible hypothesis that hydrogen and iron greatly preponderate over all the other elements combined ; the calculation of opacity for this hypothesis would require rather special treatment, which we do not here consider. We can only say that it might be appreciably lower than that obtained by applying our formulæ to iron as the mean element of a mixture.

2. We may conveniently start from *I.C.S.*, equation (156·3), which states that the emission per gram per second of radiation of frequency ν to $\nu+d\nu$ is $Q d\nu$, where Q is independent of ν and is given by

$$Q=\frac{\rho}{\mathrm{H}^2\mu A(1+f)}\frac{32\pi^2}{3\sqrt{3}}\frac{Z^2e^6}{c^3m^2V}, \tag{3}$$

the free electrons being here supposed all to have the same kinetic energy

$$E=\tfrac{1}{2}mV^2. \tag{4}$$

The guillotine factor is not included.

This result corresponds to classical theory. By quantum theory the electrons in emitting frequency ν fall to an energy level

$$\chi=E-h\nu. \tag{5}$$

Since for negative values of χ the possible energy levels are discrete, the emission, instead of being continuous, would actually be heaped up at particular values of ν. But in stellar applications this concentration to particular values can be ignored, because it is introduced only to be removed again. A reasonable mixture of elements will afford numerous possible negative levels χ without long gaps or thin places ; further, instead of a single E we have a Maxwellian distribution of E. The distribution of emission is thus rendered continuous and smooth again, and the classical formula holds approximately. The only reservation is that Q falls to zero at very high frequency, since the series of negative levels terminate with the K-levels of the atoms. This "ultimate guillotine" is placed at too high frequency to affect our present calculations appreciably ; it would, however, operate if we were attempting to calculate the opacity of hydrogen or other very light elements.

The emission $Q d\nu$ is calculated on the assumption that all the lower energy levels are vacant. If a level is already occupied, the free electron cannot drop into it, and the corresponding emission is inhibited. If in a range $d\chi$ the proportion of unoccupied levels is $x(\chi)$, the emission is reduced in the ratio $x(\chi)$, so that by (5) it is

$$x(E-h\nu)Q d\nu. \tag{6}$$

The proportion of vacant levels is given by the usual ionisation formula,

$$\frac{1-x}{x}=\tfrac{1}{2}\sigma_0\left(\frac{h^2}{2\pi mRT}\right)^{\frac{3}{2}}e^{-\chi/RT}, \tag{7}$$

where σ_0 is the density of free electrons at a place of potential zero. This formula assumes that the free electrons are non-degenerate (*i.e.* that x is nearly unity for positive levels χ). We write

$$\tfrac{1}{2}\sigma_0\left(\frac{h^2}{2\pi mRT}\right)^{\frac{3}{2}} = e^{\psi_1/RT}, \tag{8}$$

and call ψ_1 the mean ionisation level for the temperature and density considered. By (7) and (8),

$$(1-x)/x = e^{(\psi_1-\chi)/RT}, \tag{9}$$

so that levels are more than or less than half-ionised according as they are above or below ψ_1.* By (9),

$$x(\chi) = \left\{1 + e^{(\psi_1-\chi)/RT}\right\}^{-1}. \tag{10}$$

3. In my original rough calculation of g I took the levels above ψ_1 to be all vacant and those below ψ_1 to be all occupied. This exaggerates the reduction of opacity. We shall now use the continuously changing x given by (10). We have first to integrate for electrons with a Maxwellian distribution of E.

The number of free electrons with kinetic energies between E and $E+dE$ is proportional to

$$e^{-E/RT}E^{\frac{1}{2}}dE.$$

Since $Q \propto E^{-\frac{1}{2}}$ by (3), we see that the whole emission of frequency ν is reduced by insertion of the factor x in the ratio

$$\bar{x} = \int_0^\infty xe^{-E/RT}dE \div \int_0^\infty e^{-E/RT}dE. \tag{11}$$

By (5) and (10),

$$x = \frac{1}{1 + e^{(\psi_1 + h\nu - E)/RT}}.$$

Hence by (11),

$$\bar{x} = \frac{\log\left(1 + e^{(\psi_1+h\nu)/RT}\right)}{e^{(\psi_1+h\nu)/RT}}. \tag{12}$$

We have applied the reduction factor x to the emission from free-free transitions ($E > h\nu$) as well as from free-bound transitions. This is correct, because the exclusion principle, which prevents transition to occupied cells, applies to cells of positive energy as well as to cells of negative energy. But if the reduction factor for these free-free transitions is at all important, it means that the cells corresponding to positive energy are becoming congested, *i.e.* that degeneracy of the free electrons is beginning ; and other consequences of the degeneracy should then be considered.

4. By Rosseland's theory the coefficient of opacity is given by (*I.C.S.*, equation (157·15))

$$\frac{1}{k} = \frac{c}{Q}\int_0^\infty \frac{I(\nu)}{\bar{x}}\frac{\partial I(\nu)}{\partial T}d\nu \div \int_0^\infty \frac{\partial I(\nu)}{\partial T}d\nu. \tag{13}$$

* *I.C.S.*, p. 253. The factor $\frac{1}{2}$ in (7) and (8) is a recent amendment due to the recognition of electrons of opposite spin.

The guillotine factor is the factor required to correct the crude calculation in which $\bar{x}$ is omitted ; it is therefore the ratio of the values of k calculated from (13) by excluding and by including $\bar{x}$ respectively. We write

$$z = h\nu/RT\,; \quad \alpha = \psi_1/RT.$$

Then by Planck's law,

$$I(\nu)\frac{\partial I(\nu)}{\partial T} \propto \frac{z^7 e^z}{(e^z-1)^3},$$

and we obtain

$$g = \int_0^\infty \frac{e^{\alpha+z}}{\log(1+e^{\alpha+z})}\frac{z^7e^z dz}{(e^z-1)^3} \div \int_0^\infty \frac{z^7e^z dz}{(e^z-1)^3}. \qquad (14)$$

From (14) g can be calculated as a function of α or ψ_1.

The following table has been calculated by mechanical integration by Mr. H. E. Green from (14):—

Guillotine Factors

$-\psi_1/RT$	g	$\frac{2}{3}$ Weight	$-\psi_1/RT$	g	$\frac{2}{3}$ Weight
10·0	1·004	2·4–5·2	3·5	2·64	...
9·0	1·012	...	3·0	3·50	3·4–8·5
8·0	1·032	...	2·5	4·80	...
7·0	1·081	2·5–5·6	2·0	6·78	3·8–8·9
6·0	1·190	...	1·5	9·80	...
5·0	1·456	2·7–6·8	1·0	14·44	4·2–9·2
4·5	1·701	...	0·5	21·55	...
4·0	2·074	3·0–7·8	0·0	32·54	4·3–9·4

The column headed "$\frac{2}{3}$ Weight" may be of use in estimating the appropriate value of the effective nuclear charge Z, making allowance for shielding. It gives the two limits of z which include $\frac{2}{3}$ of the whole amount of the first integral in (14), $\frac{1}{6}$ being above and $\frac{1}{6}$ below this range of z. If the corresponding limits of ν are ν_1, ν_2, two-thirds of the outward flow of radiation is between frequencies ν_1 and ν_2. It is therefore especially important that our formulæ should be correct between these frequencies ; in particular we should choose Z with regard to emission at these frequencies. It might be thought that for low values of $-\psi_1$, *i.e.* for low ionisation, Z would be greatly reduced by shielding, since the captures take place mainly in the high levels of the atom ; actually the reverse happens, and a value of Z for rather deep levels should be used. The quantity which is directly calculated is the *transparency* $1/k$, so that most weight is attached to the deep levels where captures are rare.

The value $-\psi_1/RT = 0$ is an arbitrary stopping point for the table. We have not continued it to positive values of ψ_1, because the degeneracy of the electrons is already becoming appreciable at $\psi_1 = 0$.

The table replaces Table XXIX in *Internal Constitution of the Stars* (p. 233).

Addendum.—There is a small correction which I might have amalgamated with the guillotine factor if I had thought of it in time, viz. the "correction for stimulated emission." Assuming that the absorption coefficient k_ν given

by Kramers's theory is strictly accurate for weak radiation, then the absorption coefficient for radiation of equilibrium intensity at temperature T is

$$k_\nu' = k_\nu(e^z - 1)/e^z$$

where $z = h\nu/RT$. The difference $k_\nu' - k_\nu$ corresponds to Einstein's "negative absorption."

This reduction of absorption is commonly treated as if it were an increase of emission of radiation *in the same direction* (stimulated emission); but the reduction appears to be an ordinary saturation effect, and at least in astronomy there is no object in introducing the fiction of stimulated emission.

When this reduction is included in the integral in (14) we obtain an amended guillotine factor g'. It is easily seen that g'/g is a maximum when $-\psi_1/RT$ is very large, and that

$$\left(\frac{g'}{g}\right)_{max} = \int_0^\infty \frac{z^7 e^{2z} dz}{(e^z - 1)^4} \div \int_0^\infty \frac{z^7 e^z dz}{(e^z - 1)^3} = 1{\cdot}059.$$

The correction is therefore only of academic interest, since it amounts to 6 per cent. at the most. It decreases rapidly when $-\psi_1/RT$ becomes less than 5.

AN INTERPRETATION OF ABSORPTION AND EMISSION LINES IN EARLY TYPE SPECTRA.

Cecilia H. Payne, Ph.D.

The field of investigation represented by the B and O stars has been incompletely explored. General discussions of absorption spectra in the light of the theory of ionization * have led to the somewhat indefinite conclusion that the temperatures of early type stars must be high, but the colours of many of them have been shown to be incompatible with high temperature.†

Three factors combine to make a study of the spectra of these stars appropriate at the present time. First, the laboratory analysis of the spectra of elements that occur in the atmospheres of the B and O stars is now reasonably complete. Secondly, with spectra of longer dispersion becoming available through improved instrumental equipment, studies of line spectra will soon be made with greater detail; at such a moment it is fitting to summarize the best results that have hitherto been obtained with more modest dispersion. Thirdly, as noted in a previous paper,‡ the mechanical interpretation of bright lines in the spectra of early stars, recently advanced by Struve,§ has to the writer seemed inadequate; data are here presented confirming the opinion that a more complete picture can be formulated on a physical basis.

* Fowler and Milne, *M.N.*, 83, 403, 1923; 84, 499, 1924; Payne, *Harvard Monograph*, No. 1, 1925.

† *Cf.* Payne, *Harvard Monograph*, No. 3, 1930.

‡ Payne and Maulbetsch, *H.C.*, 364, 1931. § Struve, *Ap. J.*, 73, 94, 1931.

The first part of this paper is concerned with the identification and interpretation of the observed absorption lines. The second part surveys the bright line spectra that are found for B and O stars, and the conclusion is reached that Rosseland's * interpretation provides, with some modification, a satisfactory working picture of the processes involved.

1. *Sources of Data.*—Stellar data for the discussions in sections 2 and 3 are taken from the following sources: A. J. Cannon (*H.A.*, **28**, 233, 1897) for the spectra of ζ Puppis, 29 and 30 Canis Majoris, β Centauri, α Pavonis, ϕ Velorum, and, in part, ϵ and γ Orionis; Baxandall (*Publ. Sol. Phys. Comm.*, 1914) for the spectra of ι, ζ, and κ Orionis, and, in part, ϵ and γ Orionis; H. H. Plaskett (*Publ. Dom. Ap. Obs.*, **1**, 351, 1922) for 10 Lacertæ; and Lunt (*Ann. Cape Obs.*, **10**, 48B, 1912) for ϵ Canis Majoris.

The sources of data for laboratory spectra, used in making the identifications, are given in Table I.

TABLE I

Laboratory Spectra Used in Identification

Hydrogen	Fowler, *Report on Series in Line Spectra*, 1922.
Helium I	Fowler, *loc. cit.*
Helium II	Fowler, *loc. cit.*
Carbon II	Grotrian, *Graphische Darstellung der Spektren von Atomen und Ionen mit ein, zwei, und drei Valenzelektronen*, 1928.
Carbon III	Grotrian, *loc. cit.*
Nitrogen II	Fowler and Freeman, *Proc. Roy. Soc.*, A, **114**, 662, 1927; Freeman, *Proc. Roy. Soc.*, A, **124**, 654, 1929.
Nitrogen III	Freeman, *Proc. Roy. Soc.*, A, **121**, 318, 1928; Grotrian, *loc. cit.*
Nitrogen IV	Bowen, *Ap. J.*, **67**, 1, 1928; Freeman, *Proc. Roy. Soc.*, A, **127**, 330, 1930.
Oxygen II	Fowler, *Proc. Roy. Soc.*, A, **110**, 476, 1926; R. H. Fowler and Hartree, *Proc. Roy. Soc.*, A, **111**, 83, 1926; Bowen, *Phys. Rev.*, **29**, 243, 1927; Russell, *Phys. Rev.*, **31**, 27, 1928.
Oxygen III	Mihul, *C.R.*, **183**, 1035, 1926; **184**, 89, 874, 1927; Fowler, *Proc. Roy. Soc.*, A, **117**, 317, 1928; Bowen, *Phys. Rev.*, **29**, 231, 1927.
Oxygen IV	Freeman, *Proc. Roy. Soc.*, A, **127**, 330, 1930.
Magnesium II	Grotrian, *loc. cit.*
Aluminium II	Grotrian, *loc. cit.*
Aluminium III	Grotrian, *loc. cit.*
Silicon II	Grotrian, *loc. cit.*
Silicon III	Grotrian, *loc. cit.*
Silicon IV	Grotrian, *loc. cit.*
Phosphorus III	Grotrian, *loc. cit.*
Sulphur II	Ingram, *Phys. Rev.*, **32**, 172, 1928.
Sulphur III	Ingram, *Phys. Rev.*, **33**, 909, 1929.

2. *Identifications and Intensities of Lines.*—Almost all the stellar lines studied can be identified without ambiguity, in spite of undoubted inaccuracy of measurement; in this particular the hotter stars are easier to deal with than those later in the spectral sequence, which have spectra rich in lines. So complete are the identifications, and so many of the expected lines are

* Rosseland, *Ap. J.*, **63**, 218, 1926.

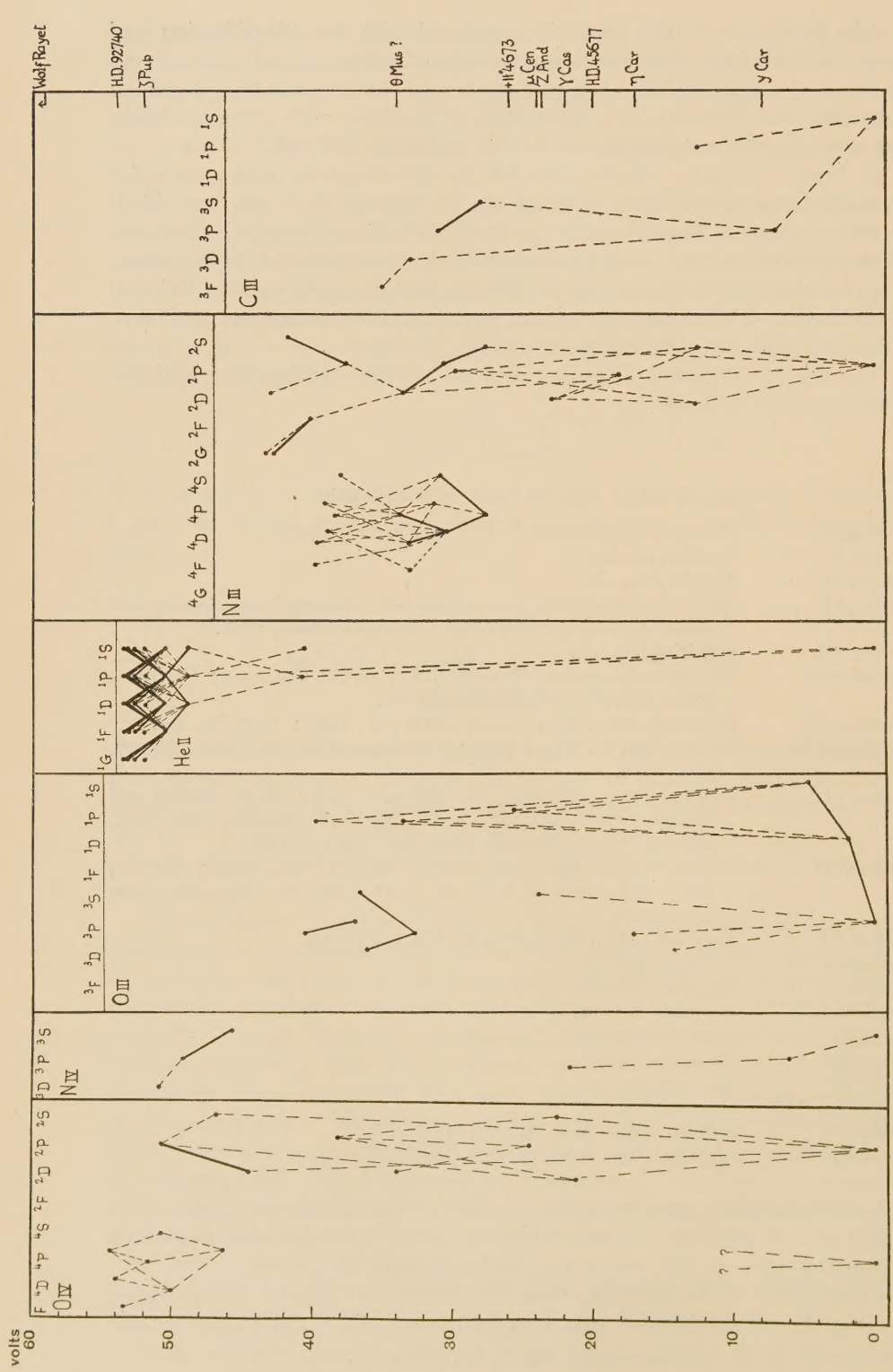

FIG. 1.—*Grotrian diagrams for the atoms of highest ionization potential found in B and O stars. Ordinates are energies in volts, reckoned from the lowest state. The line forming the upper boundary of each diagram corresponds to the ionization potential of the corresponding atom. Lines in the spectral range that can be examined astrophysically are represented by unbroken lines; if the corresponding multiplet is observed in stellar spectra the line is heavy; otherwise it is a light line. Wave-lengths outside the accessible spectrum range are represented by broken lines. At the right of the diagram the excitation levels for the bright line spectra discussed in section 6 are indicated.*

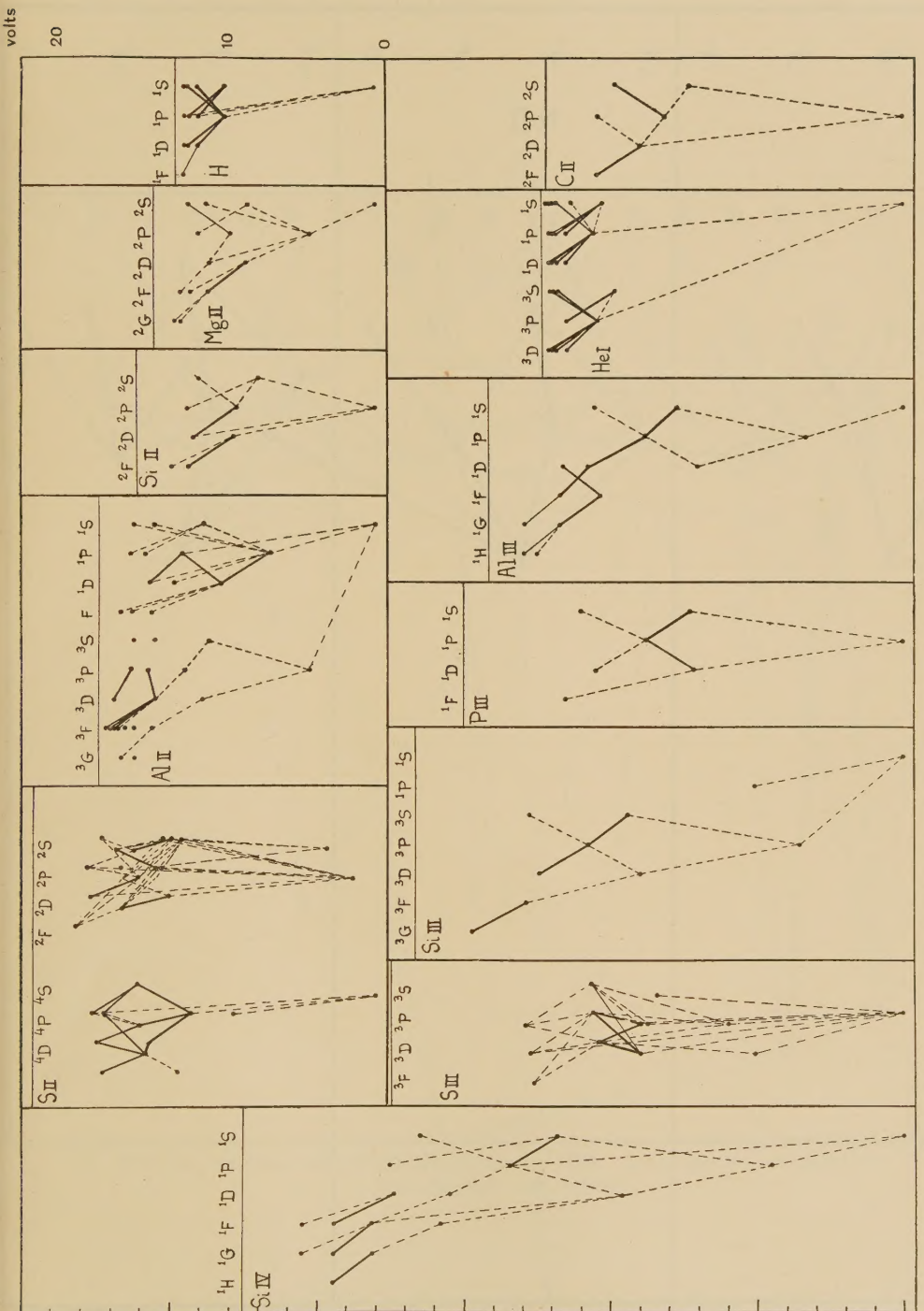

FIG. 2.—*Grotrian diagrams for atoms of ionization potential between 45 and 13 volts, recorded in stellar spectra. Symbols are the same as for fig. 1. Energies for the atoms from* Si^{+++} *to* C^{+} *are shown at the left margin; energies from* S^{+} *to* H *are shown on the right. For simplicity some of the terms in the spectra* P *III and* Si *II have been omitted. Many of the higher members of the hydrogen and helium series could not be represented, owing to the crowding of the terms.*

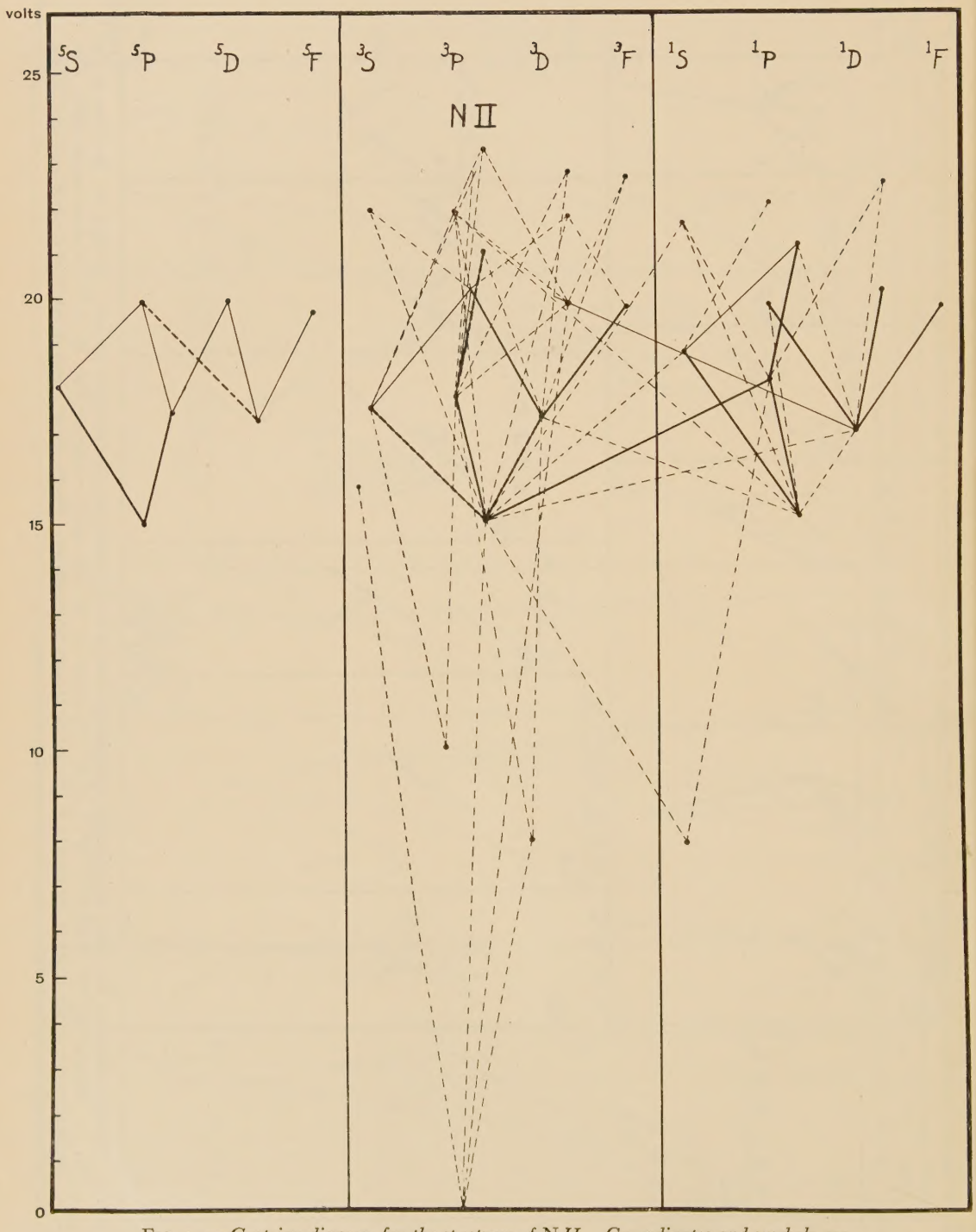

FIG. 3*a*.—*Grotrian diagram for the spectrum of* N *II. Co-ordinates and symbols are the same as for figs.* 1 *and* 2.

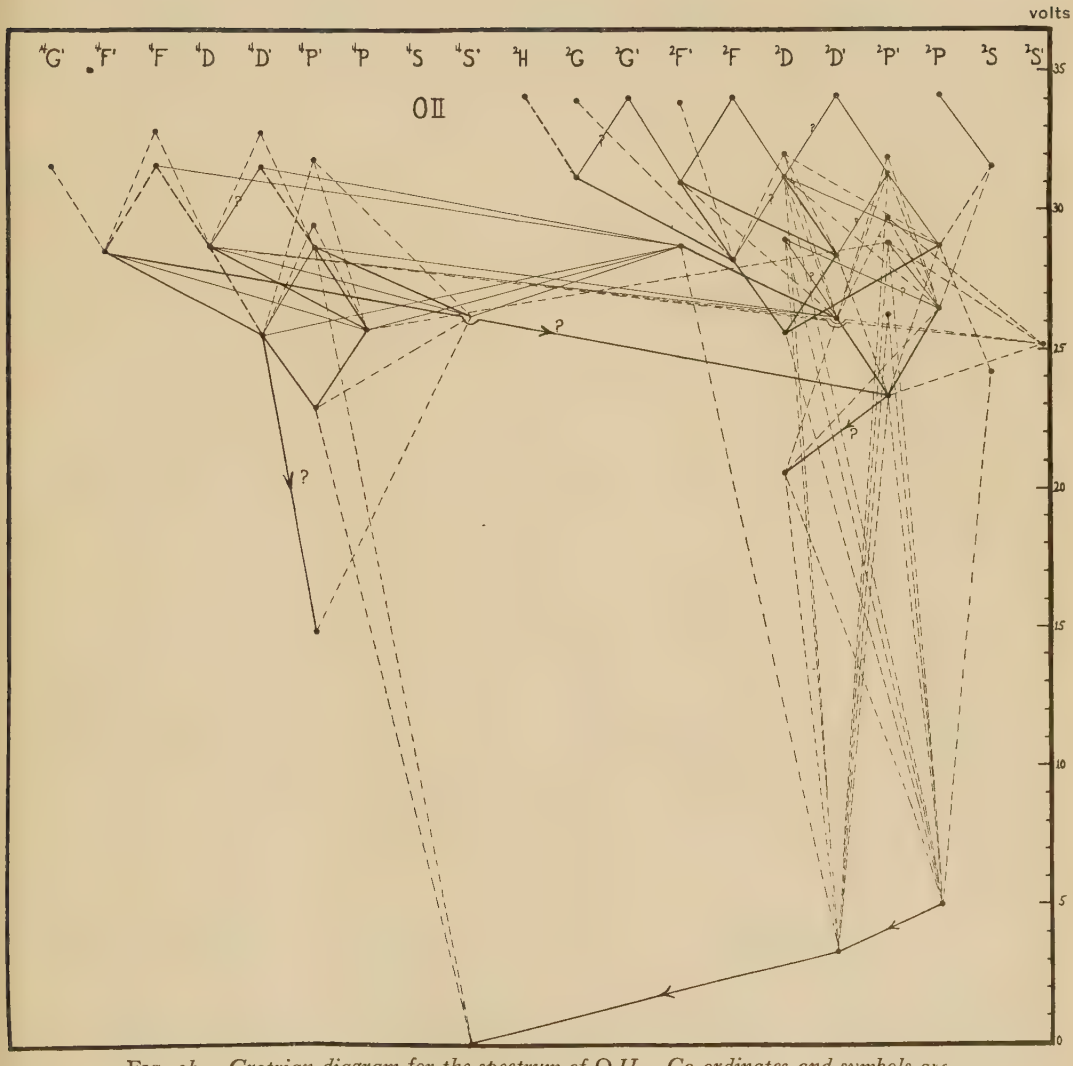

FIG. 3b.—*Grotrian diagram for the spectrum of O II. Co-ordinates and symbols are the same as for figs. 1 and 2.*

actually represented, that it seems likely that an examination with larger dispersion than has previously been available will complete the identifications with comparatively little labour.

In preparing the data for discussion the stellar intensities for the lines concerned were reduced to the same scale. The scale of *Harvard Annals*, **28**, was adopted as standard, and the scales used by Baxandall and Lunt were reduced to it empirically; Plaskett's intensity estimates were used without reduction. The relative intensities probably have an accuracy as great as can be expected from the comparison of spectra made with different instruments and types of camera; they are useful in examining changes along the sequence, as well as in determining the probability of the identifications.

3. *The Grotrian Diagrams.*—The lines in the spectra of the thirteen stars considered are nearly all satisfactorily identified. Moreover, the strong lines of all the substances which might be expected to occur in the stellar atmospheres under the conditions concerned are actually found. The complete representation of the expected lines is illustrated in figs. 1 and 2, which depict Grotrian diagrams for the spectra of all atoms that are known to be relatively common in the atmospheres of stars and that have ionization potentials greater than 20 volts. The singly ionized atoms of aluminium, silicon and magnesium, and the atom of hydrogen, are also shown in fig. 2, thus completing the list of atoms found in stars earlier than A0. The Grotrian diagrams for the more complex atoms of *N* II and *O* II are shown in figs. 3*a* and 3*b*.

The Grotrian diagrams are complete (except in a few cases, where specific terms have been omitted), and the lines corresponding to wave-lengths not in the range of stellar wave-lengths hitherto studied are broken. Heavy continuous lines denote wave-lengths identified in the stellar spectra, and light continuous lines represent the few cases in which lines, actually in the observed range, are not recorded for the stars. The unrecorded, but possible, lines are either lines arising from atomic states of relatively high excitation, or intersystem combinations. The general impression derived from figs. 1 and 2 is that the absorption spectrum for B and O stars is the normal result of high ionization.

If a line corresponding to a given excitation potential (lower state) appears in the absorption spectrum of a star, we may expect that lines requiring less excitation will also appear, unless the excitation potential in question is so far above the ionization potential of the parent atom that ionization is effectively complete. In every spectrum it is possible to pick out the line of highest excitation, and in a general way it has been found that for all the stars considered, the remainder of the spectrum is normal.

The general behaviour of the lines of various atoms is illustrated in the self-explanatory fig. 4, in which intensities of lines are plotted against spectral classes, and positions of estimated maxima are indicated. Fig. 5 shows the relation of the spectral class at which maximum is attained to the ionization potential of the corresponding atom. The data entering fig. 5 are given in Table II.

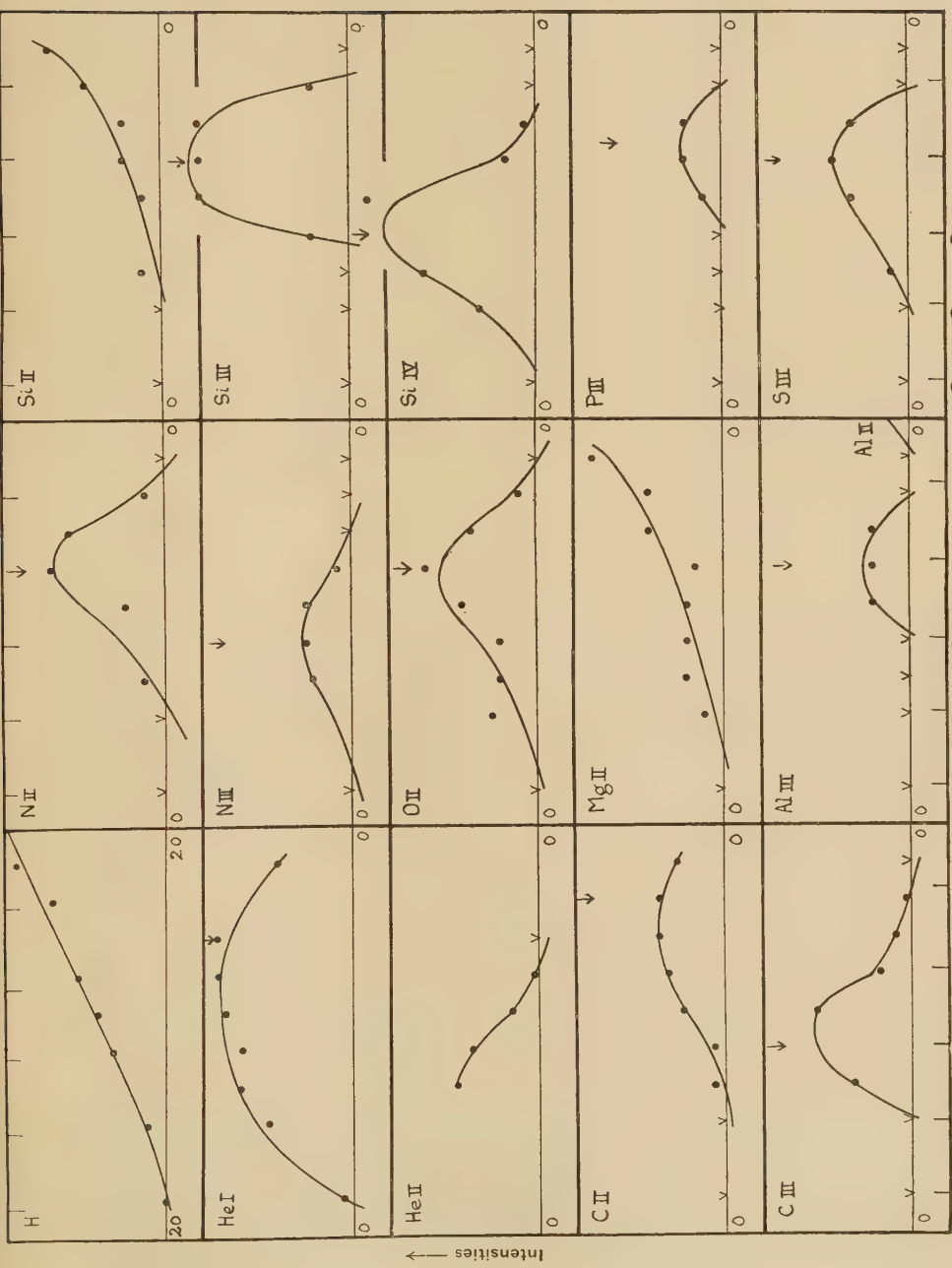

FIG. 4.—*Intensities of absorption lines of various substances in the spectra of stars of Classes B5 to O5. Ordinates are intensities (on differing scales); abscissæ are spectral classes. Arrows indicate the estimated spectral classes at which the lines have their maximum intensity. The sign V indicates that the corresponding ordinate is negative.*

TABLE II

Maxima of Intensity, and Ionization Potentials of Atoms

Spectrum	Maximum	Ionization Potential (volts)
H	A2	13·54
He I	B2	24·48
He II	none (O5)	54·16
C II	B3	24·28
C III	O9 ?	46·3
N II	B2	29·50
N III	O9	47·2
N IV	none	80·0
O II	B1	34·94
O III	none (O5)	54·5
O IV	none	76·0
Mg II	A0	14·97
Al II	B8	18·74
Al III	B1	28·3
Si II	A0	16·27
Si III	B1	33·35
Si IV	O9	44·95
P III	B1–B2	30·0
S II	B5	23·32
S III	B0	34·9

There can be no doubt that the data of Table II and fig. 5 point to an increase in ionization, and therefore in temperature, with progressively earlier spectral class. The relation of ionization potential to spectrum is much closer than for the cooler stars, and, at least for the B and O stars with absorption spectra, we may visualize a continuous progression in effective temperature and luminosity (and therefore presumably in size, surface gravity, and partial pressure).

The puzzling feature is the discrepancy between the temperatures inferred for these stars from ionization and from colour. This large problem has been discussed elsewhere; * the crucial question is: What is implied by temperature? If degree of ionization is the adopted criterion (probably implying richness of the continuous spectrum in moderately short wavelengths) the temperature must be high. If as temperature we define the factor that governs the size of the star, the luminosities of B and O stars probably imply high temperatures. Only if apparent colour is taken as the criterion of temperature is the inferred dispersion in temperature large for the B stars. That the measured colours of B stars are undoubtedly not indicators of the energy distribution in spectral regions normally inaccessible, can be inferred from the results of Trumpler † and of Miss Anger,‡ who find that the reddening is spatially localized (at least in many cases). That the energy distribution in the spectra of B stars shows a progressive,

* Payne, *Harvard Monograph*, No. 3, 1930.
† Trumpler, *L.O.B.*, 14, 154, 1930.
‡ Anger, *H.B.*, 883, 1931.

relative increase of energy in the ultra violet as early spectral classes (and high luminosities) are approached has been shown by Miss Anger in her study of the Perseus clusters ; * as she points out, the data are compatible with a continuous increase of temperature towards the earliest classes represented.

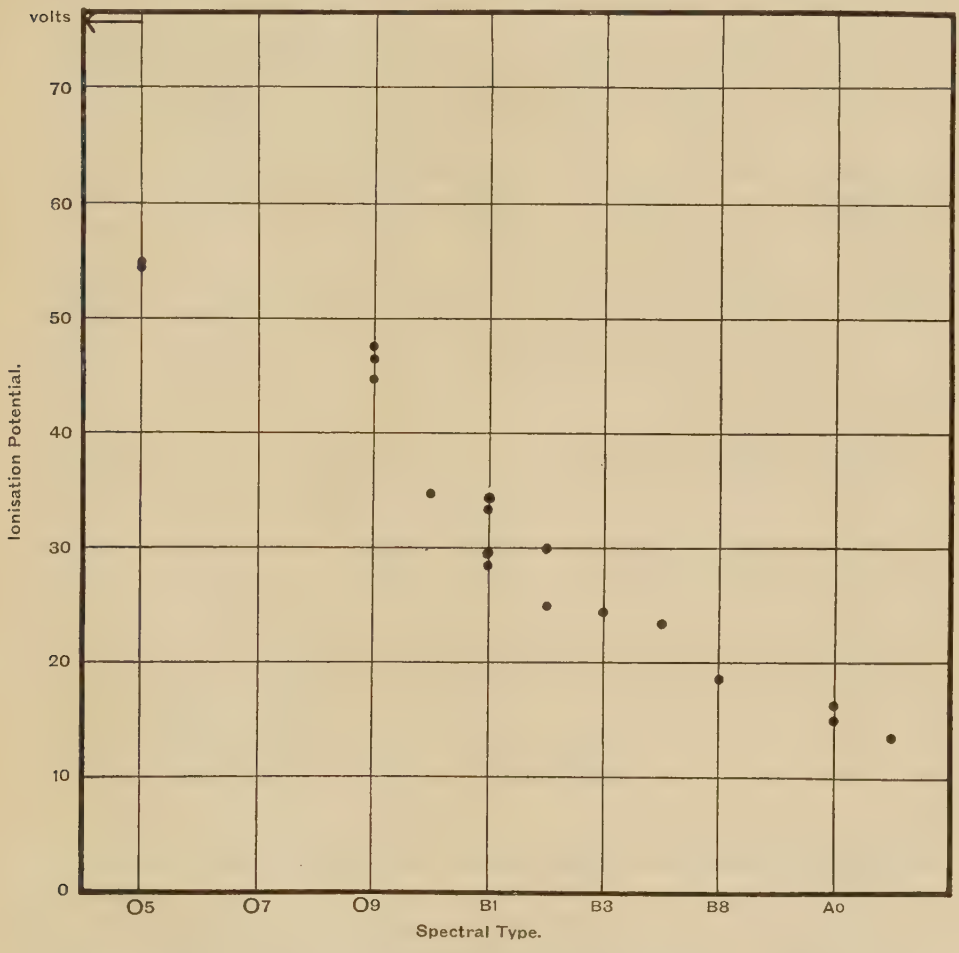

FIG. 5.—*Relation of spectral class at which lines of a given element are at maximum (abscissæ) to ionization potential of the corresponding atom, in volts (ordinates).*

The line spectra and luminosities of the B and O stars, then, correspond to high temperatures. The low temperatures postulated from the colours of many B stars are evidently only apparent (from the standpoint of the ionizing and exciting power of their radiation). The fact that the colours of some B and O stars do not correspond to black-body radiation at high temperatures points to some factor (as yet unknown) that distorts the measured energy distribution. The same factor, presumably, affects the excitation of bright line spectra, which is discussed later in the paper. The

* Anger, *H.B.*, 882, 1931.

important point is that all the major difficulties in interpreting early spectra are associated with this abnormally low colour-temperature, and, in the writer's opinion, the fact that we are ignorant of the way in which the distortion of the observed energy curve is produced should not compel us to suspend judgment on the interpretation of other phenomena, to whose elucidation it is the only bar.

4. *The Production of Bright Lines.*—One of the striking characteristics of the B and O stars is their well-known tendency to show a bright line spectrum —in general, the earlier the spectral class, the more commonly are bright lines observed.* Many speculations have been lavished on this phenomenon. As stated in an earlier paragraph, a mechanical interpretation, in terms of rapid rotation, has recently been suggested by Struve † for the bright line spectra of B stars. It seems to the writer that the quantitative data on the widths of the lines are not wholly taken into account by the rotation hypothesis; ‡ in the following pages a survey of the published data on a few typical bright line stars is presented, and an attempt is made to show that the bright line spectra can be qualitatively interpreted in terms of the simpler hypotheses of recombination and of cyclical transitions, discussed by Rosseland § under the name of fluorescence.

A brief extract from Rosseland's discussion will recall the chief points of his interpretation. ". . . In the outer atmosphere of a star conditions will be such as to favour the formation of emission lines by fluorescence. . . . As we are dealing with gases only, and in most cases with monatomic gases, the question is much simplified, as it appears to be an established fact that, for a given temperature, the appearance of fluorescent radiation is a question of pressure only. . . . The influence of pressure on fluorescence phenomena is undoubtedly due to molecular collisions. . . . During the absorption process the atoms are transferred to some higher quantum state from which, after the lapse of some time, they return to their normal state under emission of radiation. . . . The critical pressure must correspond to the case when the mean time between two successive collisions of an atom is of the same order of magnitude as the mean time interval in which the atom will remain in the quantum state from which (or in which) the emission process in question takes place." ||

"When W [the dilution factor] is less than unity . . . it is easier for the atom to complete the circuit over all states in the direction $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$ (one absorption-act and two emissions) than in the direction $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ (two absorptions and one emission). The effect of the atoms on the stream of radiation will, therefore, be to create one absorption line, corresponding to the transition $1 \rightarrow 3$, and two emission lines, corresponding to the transitions $3 \rightarrow 2$, $2 \rightarrow 1$. This result illustrates clearly the general process of generation of emission lines by fluorescence." ¶

* See *Harvard Monograph*, No. 3, 1930.

† Struve, *loc. cit.*, 1931.

‡ Payne and Maulbetsch, *loc. cit.*, 1931.

§ Rosseland, *loc. cit.*

|| Rosseland, *loc. cit.*, 223, 1926.

¶ Rosseland, *loc. cit.*, 231, 1926. In a later publication Rosseland has remarked: " Der Ursprung der Emissionslinien in O- and B-Spektren liegt noch völlig im

It will be noted that this interpretation covers not only the production of normal bright lines, but also that of the forbidden lines, with the added proviso, made by Eddington,* that the radiation must be sufficiently dilute.

A detailed practical application of Rosseland's suggestion was made by H. H. Plaskett for the spectrum of the stellar component of Z Andromedæ.† Plaskett concluded that for this star "though extension of the atmosphere favours the appearance of emission lines in the integrated spectrum of photosphere and atmosphere, some additional unknown factor is required to account for the great luminosity of these emission lines."

In Plaskett's extension of the argument to other bright line stars of early type he reaches the conclusion that "for the . . . ordinary Be stars . . . and absorption O's with faint emission, it is probable that extension of the stellar atmosphere is a necessary and sufficient condition for the appearance of emission lines. . . . For the . . . very heterogeneous class, which includes many bright iron line stars, Wolf-Rayet stars and late type variables, it is clear that factors other than, or, in the case of Z star at least, in addition to atmospheric extension are responsible for the appearance of the emission lines."

As for the heterogeneous class of stars mentioned by Plaskett, the Wolf-Rayet stars can probably be included in Rosseland's explanation, and the low-temperature bright line stars (such as the stellar component of Z Andromedæ) can be interpreted by it, except that we do not know the source of the high excitation to which they are obviously subject. The variable stars of late type are of a different character, and the present paper does not consider them. It is the purpose of the succeeding argument to show that for bright line stars of spectral classes earlier than A, the bright lines may be produced by recombination and "fluorescence"; but for some stars we are ignorant of the conditions that produce a spectral energy distribution corresponding to a temperature much lower than that implied by the observed absorption and emission spectra. The nature of this condition must be regarded as a separate problem.

5. *General Conditions for the Production of Bright Lines.*—In order to emit a bright line, an atom in a stellar atmosphere must arrive at the upper state corresponding to the line, and must do so by a route other than the absorption of the line in question. This may be effected by (*a*) excitation from a low state by radiation of short wave-length; (*b*) ionization. In the latter case the atom produces a recombination spectrum. The raising of the atom to the high state is of course favoured by high temperature at the source; for the fall to the lower state the conditions must be such that the atom does not lose its energy more readily in other ways: thus the production of bright lines is a function of temperature, of pressure, and of dilution

Dunkeln . . . Man muss sich . . . vorstellen, dass der Stern von einer sehr ausgedehnten und dünnen Wasserstoffhülle umgeben ist, in der Fluoreszenzerscheinungen sich abspielen. . . . Möglicherweise sind aber die hellen Linien mit Fleckenaktivität verbunden . . ." (Rosseland, *Physik der Sternatmosphäre*, 202, 1931).

* Eddington, *M.N.*, 88, 134, 1927.

† H. H. Plaskett, *Publ. Dom. Ap. Obs.*, 4, 157, 1928.

of radiation. It is readily seen why bright lines are commonest in the hotter stars, and in the atmospheres of the most luminous of these.

Ionization and excitation depend directly on the temperature of the incident radiation. The pressure alone determines the rate of recombination, and thus, by determining the degree of ionization of the atoms, it governs the number of a given kind which are available for the particular transition under consideration. Thus *Si* IV is more likely to occur as an emission spectrum in Class O7 than in Class B1, because there are actually more trebly-ionized silicon atoms in the atmospheres of the former.

It should be borne in mind that the effects of pressure and dilution of radiation will differ for different atoms in a way that depends on the transition coefficients for the particular states concerned; a pressure that suppresses the bright lines coming from one atom may spare the emission spectrum of another.*

When the spectrum of a star showing bright lines is analysed, it may be assumed that the exciting conditions are adequate to raise the atom considered to the upper state of the line of highest excitation observed. The interpretation just suggested can be considered satisfactory if the following conditions are met:—

(*a*) Lines arising from levels whose ionization potentials are lower than the observed maximum excitation also appear as bright lines, unless their ionization potentials are so much lower than the limiting excitation potential that the atoms are effectively and completely ionized. These bright lines are regarded as produced by recombination, and the higher is the corresponding ionization potential, the more commonly do they appear. Such bright line spectra contain all the lines of the atom in question that fall in the observable range.

(*b*) Lines arising from upper levels that are lower than the limiting level for the bright line of greatest excitation, but which come from atoms of ionization potential higher than the limiting excitation, will appear, in certain circumstances, as bright lines. Recombination will not in general be responsible for these lines, as the conditions are inadequate to ionize the corresponding atoms, and "fluorescence" (raising of the atom from the low state by one absorption to an excited state, and relapse by two or more jumps, with the emission of radiation, to the low state) must be responsible. Therefore only such bright lines will be expected as arise from an upper state that combines directly with a low state of the atom, or that can be reached from such an upper state with the emission of energy; others may occur, but they should be relatively weak. This limitation accounts, as will be shown, for the otherwise puzzling absence from bright line spectra of some relatively strong lines.

The forbidden lines in many stellar sources may occur under circumstances (*a*) or (*b*), the conditions necessary for their production being low pressure, dilution of radiation, and transportation to the corresponding upper state.

In the next section a number of bright line spectra will be reviewed,

* *Cf.* Bowen, *P.N.A.S.*, 14, 30, 1928.

and analysed as to whether they satisfy condition (*a*) or (*b*) or neither. Merrill has analysed the bright line spectra of +11°4673, referred to later, and of other stars, on the basis of excitation energies, in a way similar to that followed below.

6. *Analysis of Selected Bright Line Spectra.*—The stars chosen for illustration are shown in Table III. The entries are arranged roughly in order of decreasing temperature; the inclusion of a Wolf-Rayet star represents a rather daring extrapolation.

TABLE III

Bright Line Stars Selected as Typical Examples

Spectral Class	Star	Reference
Oc	H.D. 92740	Payne, *H.B.*, 843, 1927.
O5e	ζ Puppis	Cannon, *H.A.*, 28, 233, 1897.
Wolf-Rayet	...	Beals, *Publ. Dom. Ap. Obs.*, 4, 275, 1930.
O9w	θ Muscæ	Payne, *H.B.*, 842, 1927.
Boe	γ Cassiopeiæ	Curtiss, *Publ. Mich. Obs.*, 1, 2, 1916.
B3e	μ Centauri	Cannon, *H.A.*, 28, 178, 1897; Lockyer and Baxandall, *Proc. Roy. Soc.*, 74, 548, 1905; Gerasimovič, *H.B.*, 854, 1928.
cF5	ι_1 Scorpii	Davidovitch, *H.B.*, 835, 1927; 846, 1928.
Pec.	η Carinæ	Merrill, *Ap. J.*, 67, 392, 1928.
Pec.	H.D. 45677	Merrill, *Ap. J.*, 67, 405, 1928.
Pec.	+11°4673	Merrill, *Mt. W. Contr.*, 381, 1929.
Pec.	Z Andromedæ	H. H. Plaskett, *Publ. Dom. Ap. Obs.*, 4, 119, 1928.

The typical stars will now be briefly described, as far as the relevant features of the spectrum are concerned.

H.D. 92740.—The Pickering and Fowler series of *He* II, the Balmer series, lines of *Si* IV, and the lines of *N* III and *C* III at 4640, all appear more or less in emission. The only wholly dark lines are the (presumably stationary) H and K lines of ionized calcium. The excitation corresponds to at least 54 volts (Pickering series), so that we must visualise the spectra, *Si* IV, *N* III and *H*, as formed on recombination. The spectra mentioned (excepting *H*, *O* III, *O* IV and *N* IV) are the only ones in Table II with ionization potentials greater than 40 volts; hence they may be regarded as the ones most likely to appear under such conditions of excitation.

ζ PUPPIS.—The Fowler *He* II lines and the *N* III lines at 4640 are the only ones that appear in emission. The Pickering lines and the Balmer lines form the absorption spectrum; from the relative intensity of the Balmer and Pickering lines (each of the former being superimposed on a component of the latter) we infer that hydrogen contributes very little to the spectrum. The excitation must be at least 51 and less than 53 volts, as the Pickering series appears in absorption only.

In comparing ζ Puppis and H.D. 92740, it seems as if the former were the more highly ionized, the latter the more highly excited (judging from the relative strength of the Balmer and Pickering lines, and the absence of *Si* IV from ζ Puppis). The difference is compatible with the suggestion

that H.D. 92740 has the lower temperature and the higher surface gravity —a suggestion of which no direct test is possible. The required difference in temperature is small, as the necessary difference of excitation need not be greater than the difference of 3 volts between the upper states of the Pickering and Fowler series.

It would be of the greatest interest to know whether either of these O stars shows the lines of *O* IV and *N* IV; if so, the lines would be produced (in the present case) by "fluorescence," as the excitation, at least for ζ Puppis, would be inadequate to ionize *O* IV and *N* IV. Such production of the lines would imply that the upper state was reached directly from the low state of the corresponding atom. As may be seen from fig. 1, this is probable for *N* IV, less likely for *O* IV.

Wolf-Rayet Stars.—In his general summary of the bright bands for Wolf-Rayet stars, Beals * enumerates the identifications, and includes the *O* IV and *N* IV lines shown in fig. 1. A guess may be hazarded that all other lines are produced by recombination in the stars of class OW6, which is thus the most highly excited spectral class recorded. There is no direct evidence as to the relative excitation of the Wolf-Rayet stars and the two absorption O stars just noted, but general considerations suggest that the former are the more highly excited.

θ Muscæ.—The only strong emission in the spectrum of this star is that near 4640; the Fowler series of ionized helium, if present, is very faint. Therefore the excitation must be less than 51 volts and more than 31 volts (assuming the carrier to be C^{++}). If the carrier is N^{++}, the limit is 35 volts. As the ionization potential of C^{++} is 47 volts, it seems probable that the bright lines near 4640 are produced largely by excitation.

Bright Line B Stars.—(*a*) Merope is a typical bright line B5 star that shows only the emission lines of hydrogen; a B3 star with the same peculiarity is ζ Tauri. In the spectrum of the former, the intensity of the bright components falls off rapidly with decreasing wave-length; for the latter, the decrease is less rapid. Merrill has pointed out † that this difference is typical of the difference between early and late Be stars; evidently the emission spectra of the former are produced preponderantly by recombination, the latter preponderantly by emission from excited states, and this inference is in harmony with the idea that the early B stars are at a higher temperature than those of later class.

(*b*) A number of Be stars have bright metallic lines in addition to the bright hydrogen spectrum.‡ The most conspicuous lines are those of *Fe* II, though lines of *Ti* II and *Cr* II also appear. If interpreted in terms of the scheme of section 5, the iron lines represent condition (*a*), being probably produced chiefly by recombination; the ionization potential of Fe^{+} is over 16 volts; and the existence of the dark helium spectrum indicates an excitation of more than 21 and less than 24 volts.

One of the most complete investigations of the spectrum of a normal Be

* Beals, *Publ. Dom. Ap. Obs.*, 4, 271, 1930.

† Merrill, *Ap. J.*, 73, 360 (footnote), 1931.

‡ Merrill, *Mt. W. Contr.*, 334, 1927; Gerasimovič, *H.B.*, 851, 1927.

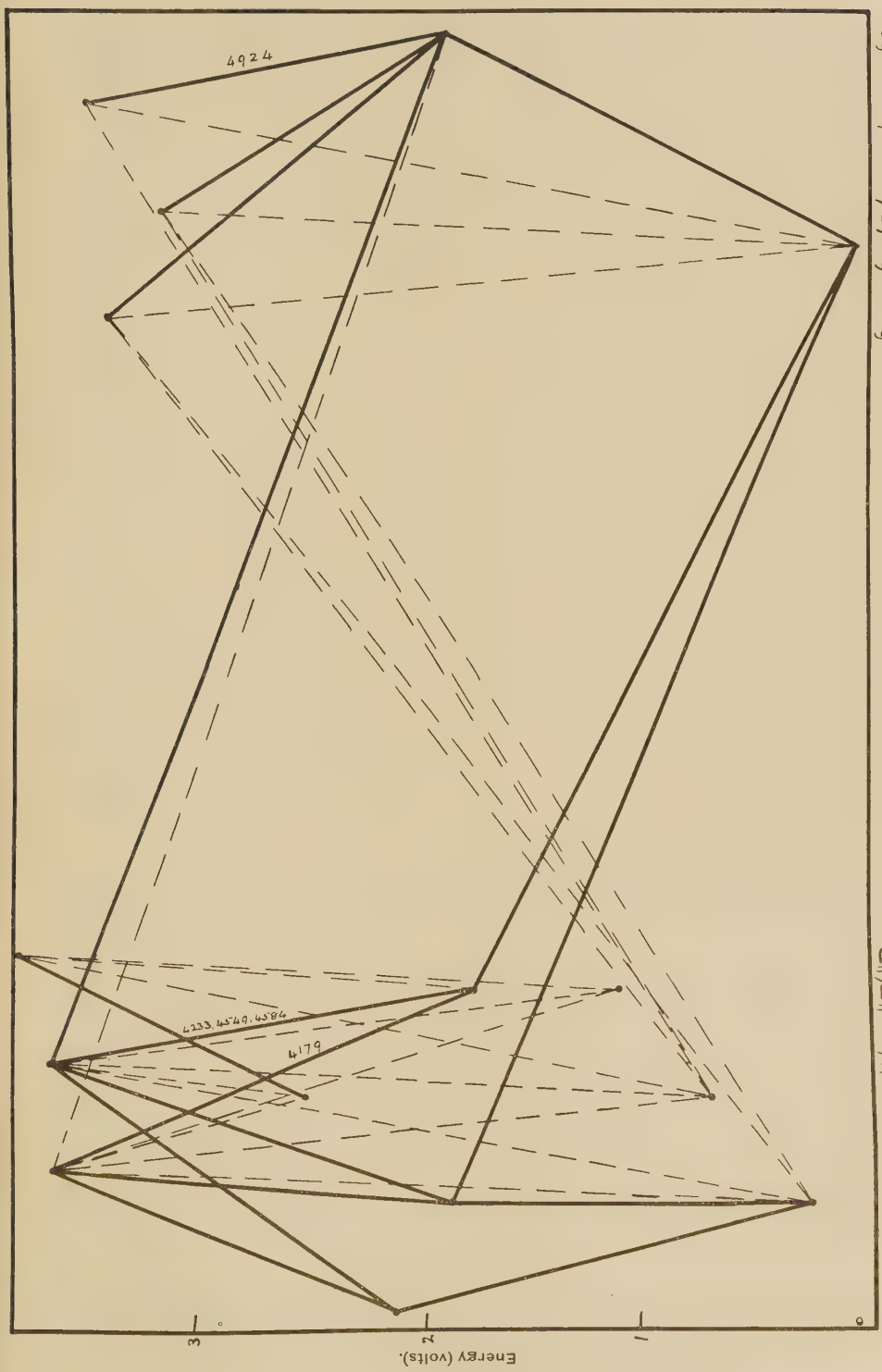

FIG. 6.—*Simplified Grotrian diagram for the* Fe^{+} *atom. Symbols are as in figs.* 1 *and* 2. *The five heavy lines representing transitions to the two low states of the atom correspond to the forbidden jumps. The ionization potential of the atom is* 16·5 *volts.*

star, showing the bright lines of ionized iron, is that made by Curtiss * for γ Cassiopeiæ. The stellar bright line spectrum is shown in the simplified Grotrian diagram for *Fe* II given in fig. 6; the atoms of *Ti* II and *Cr* II, excited in the same circumstances, are also represented by emission spectra. None of the forbidden lines of *Fe* II appears in the spectrum of γ Cassiopeiæ; we infer that the pressure is too great.

A glance at fig. 2 will elucidate the absence of other bright lines from the spectrum. The excitation may be taken to correspond to 22 volts; this excitation is inadequate to ionize the atoms of *S* III, *Si* III, *P* III, *Al* III and *C* II, so that they cannot give recombination spectra; none of these atoms has, in the accessible region, lines whose upper state combines with the low state of the atom, and therefore, though all of them have spectral lines of the requisite excitation in the region studied, none can appear in the emission spectrum. If the lines of these atoms occur in emission, they must be produced by recombination, and bright helium lines will then also be expected. It seems as if atoms of ionization potential less than 22 volts might be too far ionized to give bright recombination spectra; the lines of *S* II and *Al* II are usually weak and might escape detection; and the atoms of *Si* II, which is generally conspicuous in B stars of later class, are evidently too far ionized to produce perceptible absorption or emission.

That hydrogen, with an even lower ionization potential, can still give bright lines, may be explained in several ways. It is more abundant in stellar atmospheres than any other substance; it might have a greater tendency than most substances to form an extensive low-pressure atmosphere; and it can be ionized but once, so that an ionized hydrogen atom is always equally accessible to recombination, whereas a silicon atom, under similar conditions, may have parted with three electrons, which must be successively replaced before it can produce the spectrum *Si* I—a contingency so remote that that spectrum is unknown in the hotter stars.

The spectrum of μ Centauri (Class B3e), before the bright lines disappeared,† was very similar to that of γ Cassiopeiæ. When the bright lines were present, those of hydrogen and *Fe* II were conspicuous; moreover, the lines of *He* I had a slight tendency to emission, so that the excitation must have been about 24 volts. All the bright lines disappeared from the spectrum together, a phenomenon not within the province of the present note, though we may mention that an increase in pressure would be more effective in suppressing the bright lines than any probable decrease in temperature. The excitation of the *Fe* II lines in the upper state is comparatively low (less than 5 volts), and the absence of the lines from the spectra of most B stars is evidently not a matter of inadequate energy supply, but of too great pressure.

Bright Lines in Second Type Supergiants.—In comparing the spectra of ι_1 Scorpii and y Carinæ with that of Nova Pictoris, Davidovich ‡ noted

* Curtiss, *Publ. Obs. Mich.*, 2, 1, 1916.

† Cannon, *H.A.*, 28, 178, 1897; Lockyer and Baxandall, *Proc. Roy. Soc.*, 74, 548, 1905; Gerasimovič, *H.B.*, 854, 1928.

‡ Davidovich, *H.B.*, 835, 1927; 846, 1928.

that the $^6P - {}^6S$ lines of ionized iron were slightly bright in the two former. No other bright lines have been recorded for these stars. We have here a clear case of "fluorescence," for the available excitation is evidently not great enough to produce bright hydrogen lines (requiring about 12 volts), and the ionization potential of Fe^+ is 16·5 volts. It is noteworthy that the observed lines are reached from the lowest state of the Fe^+ atom (see fig. 6). The two stars are both of Class cF5, and their spectra indicate that they are of exceptionally great brightness, so that the existence in their atmospheres of the necessary low pressure is probable on general grounds.

All the stars whose spectra have been described fall in the category for which, according to Plaskett, Rosseland's process provides a satisfactory interpretation. The stars now to be considered have very peculiar spectra, consisting chiefly of emission lines, and for these Plaskett suggested that some additional factor was responsible. Without attempting to name this factor, it will now be shown that the composition of the spectra is compatible with a fluorescent origin.

η Carinæ.—This remarkable spectrum, consisting now only of bright lines, has been shown by Merrill * to contain the lines of hydrogen, *Fe* II (including the forbidden lines), *Ti* II and *Cr* II. Therefore the excitation must be at least 13 volts, but less than 24 volts, for no helium is represented. There is nothing to differentiate fluorescence from recombination for the iron spectrum, but the former process is possible, for all the lines that occur arise from upper levels that combine with the two lower states (one quartet and one sextet). All the forbidden lines connecting the lower states, reached by the normal lines, to the two lowest states are also represented in the spectrum.

Presumably the bright lines of *Ti* II and *Cr* II are formed in the same way. If there are forbidden lines of *Ti* II similar to those of *Fe* II, they must be of such high wave-lengths as to be outside the range of the measured spectrum. The pressure in the atmosphere of η Carinæ must be extremely low.

If the lines that normally occur in absorption for F stars are reviewed, in the same manner as those represented in figs. 1 to 3, it is evident that no other lines would be expected in the spectrum of η Carinæ. Neutral metals must be too far ionized to appear; *Mg* II is probably of too high an ionization potential to appear on recombination, and the upper state for the line 4481 does not combine directly with a low state of the atom, so that the line cannot appear by fluorescence. Similar remarks apply to other atoms of higher ionization potential. The atoms of *Ca* II, *Sr* II and *Ba* II might be expected to emit their $2P - 3D$ lines, but all are too far toward the red to have been recorded; the first may possibly be detected in the future. Probably the lines of η Carinæ, still unidentified, will at some time be ascribed to ionized metals.

H.D. 45677.—Merrill † has analysed the spectrum of another bright

* Merrill, *Ap. J.*, 67, 392, 1928.
† Merrill, *ibid.*, 67, 405, 1928.

line star, which combines an absorption spectrum with a bright line spectrum not unlike that of η Carinæ. The dark lines are those of *He* I and *Mg* II, so that the excitation must be about 20 volts. The bright lines are ascribed to hydrogen and ionized iron, and must all be formed by recombination. The presence of forbidden lines, as for η Carinæ, points to very low pressure.

+11°4673.—A very complete investigation of the complexities of this spectrum has been made by Merrill.* Without discussing the velocity data, a summary of the occurrence of the bright lines will now be made, and examined in the light of section 5. The bright lines are given in Table IV.

TABLE IV

Bright Line Spectra given by Merrill in +11°4673

Spectrum	Remarks
H	Present
He I	Present
He II	Absent
C II	Faint
N II	Forbidden lines
O II	Absent
Mg II	Faint
Al II	Present
Al III	Present
Si II	Present
Si III	Present
Si IV	Absorption only
Fe II	Forbidden lines

The highest excitation represented in emission is that of the upper state of the *He* atom, about 24 volts. The *Si* III lines, $2^3S - 2^3P$, energy of the upper state 21 volts, are the most highly excited silicon lines to appear in emission; the $2^2S - 2^2P$ lines of *Si* IV, which require only 27 volts for their stimulation, appear only as dark lines. From figs. 2 and 3 we can determine what other lines may be expected under these conditions of excitation.

The spectrum *O* II is not found in +11°4673. The ionization potential of the corresponding atom is 35 volts, so that the spectrum could not appear on recombination. A glance at fig. 3 will show that no astrophysically accessible lines have their upper states in direct combination with the low state of the atom. Hence, although the excitation potentials of many of the lines are within the limit (24 volts), the spectrum does not appear in emission.

The spectrum *S* III presumably could not form on recombination, as the ionization potential of the atom is 30 volts: the only line to be expected is at 4253·59 ($4p$D – $4s$P), reached from the $4d$D state, which combines directly with the low term; this line seems to be present in Merrill's table at 4254·1, intensity 0, though he did not identify it.

The only line of *Si* III that is of low enough excitation to appear actually

* Merrill, *Mt. W. Contr.*, 381, 1929.

does so; it cannot be formed on recombination, as the ionization potential of the atom is 33 volts, and there is no known combination of the 3^3S term with the lowest state, but the 2^3S term does so combine, and the required transition probably takes place; the wave-length involved is very small.

Evidently the relevant lines of *P* III cannot appear either on recombination or in fluorescence.

The spectrum *N* II is observed in this star, and is presumably formed on recombination; the ionization potential is 29 volts, which may account for the weakness of the lines—relatively few atoms are being ionized; the condition is very close to that for *Al* III. There is one group ($3dD - 3pD$) whose upper state combines directly with the low state of the atom, and it is, as expected, present in the spectrum. The occurrence of the forbidden lines of *N* II is an indication of low pressure.

Merrill identifies the *Al* III lines, 4D – 5F, 4P – 4D, 4S – 4P, and 4F – 5D, in the spectrum. Except the 4G – 5F lines, these comprise all the *Al* III lines that have been astrophysically observed. The excitation potential of the upper state is about 23 volts; the lines requiring an upper state excitation of about 26 volts do not appear. Possibly the spectrum may be formed on recombination; otherwise the route by which the atoms reach the 5F and 5D states is not obvious.

The spectrum *C* II would normally form by recombination, and the corresponding lines are actually found. The lines of *Al* II probably form normally on recombination; all the astrophysically observed lines are represented. The lines of *Si* II ($2^2P - 3^2D$ and $2^2D - 3^2F$) are still represented, as would be expected if they are formed by recombination. The 4481 line of *Mg* II appears under the same conditions. The Balmer series is represented, as one would expect.

The appearance of the ionized metallic spectra receives the same interpretation as that given for the two previous stars. The forbidden *Fe* II lines indicate that the pressure must be very low, a point noted by Merrill.

The absence of other lines is in accordance with expectation and will not be analysed specifically.

Z Andromedæ (Stellar Component).—The spectrum, of very similar type to that of +11°4673, will be discussed very briefly.* Helium is present, so that the excitation must be considerable, but there are several features that suggest that it is less than in the former star.

(*a*) Both *N* II and *O* II are wholly absent from the emission spectrum. It is suggested that the *N* II spectrum, like that of *O* II, could only appear in bright lines by recombination, since no astrophysically observable lines arise from terms that can be reached from the low state; but the excitation is inadequate to ionize the atoms sufficiently to give a recombination spectrum.

(*b*) In the spectrum *Si* II the lines corresponding to the D – P transition occur, but those corresponding to the F – D transition (normally the strongest) do not. The former lines arise from an upper state that combines directly with the lowest state of the atom.

* H. H. Plaskett, *Publ. Dom. Ap. Obs.*, 4, 119, 1928.

The occurrence of the spectra of ionized metals, with the forbidden iron lines, is similar to that for the last three stars discussed. None of the other lines noted by Plaskett as missing would be expected to occur on the basis of section 5.

7. *Summary of Results of Section* 6.—The origin of the bright line spectra of all the stars discussed can be satisfactorily pictured in terms of fluorescence. The O stars have excitation values corresponding to about 55 volts; the bright line B stars, about 20 volts; and certain bright line F stars show lines that could arise under not more than 5 volts. In every case a certain low pressure, specific for the atom in question, is required before the spectrum can appear. Some of the more peculiar stars discussed show the forbidden lines of *Fe* II and *N* II, requiring exceedingly low pressures for their production.

It is possible not only to account for the occurrence of all the lines observed, but also to show that no other lines would be expected.

8. *Difficulties encountered by the Interpretation.*—In his monograph on Z Andromedæ, Plaskett * pointed out that the observed brightness of the emission lines could not be accounted for quantitatively by fluorescence if the temperature of the star were 5200°, as measured by him from the energy distribution in the continuous background. This difficulty is not confined to Z Andromedæ; there are indications that η Carinæ is a star of similar energy temperature.

The present paper makes no attempt to meet these points. Reasons for carrying out the investigation in spite of not being able to do so are given in section 4. That the excitation manifested in the spectra discussed in section 6 actually occurs is certain. The writer is inclined to the opinion that it is not necessary to know the source of the excitation in order to trace the general picture of what is going on in the stellar atmospheres.

Another matter that must ultimately be covered by the theory is the distribution of energy in an emission line. Higgs † has shown that the contours of bright *Fe* II lines are similar to those of hydrogen for certain bright line B stars. At present our knowledge of bright lines is in the same state as our knowledge of absorption lines: we have a general idea of the process of formation, but no very specific picture; and in neither case is there a completely satisfactory theory of the origin of the observed contour.

1931 *August* 22.

* H. H. Plaskett, *loc. cit.*

† Higgs, *Ap. J.*, 70, 251, 1929.

NOTE ON SOME PECULIARITIES IN STELLAR SPECTRA OF TYPE B.

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(Communicated by the Director of the Norman Lockyer Observatory)

During the examination of stellar spectra taken at the Norman Lockyer Observatory for the purpose of classification or spectroscopic parallax work, some peculiarities were noted in a number of B-type spectra which made their exact classification in spectral type rather uncertain. The scheme of classification used at this observatory is based entirely on the Harvard system, using the same criteria of line intensities but with the introduction of the intermediate sub-types B4, B6, and B7.* It has, however, been occasionally found that the various criteria by which the classification is judged do not agree for a particular star owing to the abnormal behaviour of one or more lines. The *He* lines, for example, might be typical of type B8, while Fe^+ lines more nearly approach A2. The purpose of this note is to draw attention to stars in which such peculiarities have been observed.

It has been found that these stars may be arranged for convenience of description into five groups, although such classification must be regarded as only approximate. The stars within any one group show certain variations among themselves, and in border-line cases (especially if the negatives are not of good quality) there may be some uncertainty as to the group into which they should be placed. The first three groups include only stars of types B5 to B9, and refer to the behaviour of the [K] line of Ca^+ and to some of the stronger metallic lines common to A-type stars. Group IV includes only types B1 to B3, in which the O^+ lines are abnormally strong. Group V contains stars in which considerable differences exist between the Harvard classification and that determined at the Norman Lockyer Observatory. This includes stars of all the sub-groups of type B. One factor which appears (with few exceptions) to be common to all these peculiar stars is that in general they have narrow sharp lines. Their line-character classification at the Norman Lockyer Observatory varies from *ns* to *ss*, with a large proportion of *s* and *ss* stars ; very few *n* or *nn* stars are included. This suggests that they are all stars of high luminosity.

Group I : The line [K] strong for type.—In these stars the *He* lines are sufficiently strong to place the spectra in types B5 to B9, but the line [K] is as strong as in type A0 or A2. Other A-type lines which are usually associated with strong [K], such as those of Fe^+, Ti^+, and Sr^+, are weak or absent. This absence may occasionally be due simply to the poor quality of the negative, and in such cases the stars should be placed in Group II. In general, however, the negatives are sufficiently good to confirm the relative faintness of these lines. A list of stars in this group is given in Table I.

* *Monthly Notices*, 87, 366, 1927.

TABLE I

([*K*] *line strong for type*)

H.D.	Name	1900 R.A. h m	1900 Dec. ° ′	*m*	Sp.
6118	σ Psc	0 57·3	+31 16	5·46	B9
6960	HR 342	1 5·0	+63 40	5·46	B9
19065	HR 922	2 58·9	+63 41	5·82	B9
38899	134 Tau	5 43·9	+12 37	4·92	B9
113797	14 CVn	13 1·1	+36 20	5·11	B9
149212	15 Dra	16 28·2	+68 59	4·98	B8*p*
155328	HR 6383	17 5·8	+50 58	6·28	B9
164353	67 Oph	17 55·6	+ 2 56	3·92	B5*p*
175156	HR 7119	18 49·0	−15 44	5·04	B5
175863	+59° 1929	18 52·3	+59 53	6·94	B5
191243	HR 7699	20 3·8	+34 8	6·07	B8
192907	κ Cep	20 12·3	+77 25	4·40	B9
193964	71 Dra	20 17·9	+61 56	5·61	B9
196867	α Del	20 35·0	+15 34	3·86	B8
205551	HR 8259	21 31·0	+51 15	5·96	B9
208501	13 (μ) Cep	21 51·5	+56 8	6·01	B9*p*
224544	HR 9068	23 53·7	+31 49	6·36	B5

Notes to Table I

6118. Si^+ double also rather strong ; *He* weak. Possibly type Ao.
164353. Very sharp lines. H.D.C. remarks : " The lines are narrow."
149212. H.D.C. remarks : " The line K is strong and the spectrum resembles that of β Orionis except that the lines are not so narrow as in that spectrum."
191243. Very sharp lines.
196867. H.D.C. remarks : " The spectrum is slightly peculiar. The line K is as strong as in class Ao, and other solar lines are strong."
208501. Very sharp lines. H.D.C. remarks : " The lines are narrow."

Group II : A-type lines strong.—The stars in this group resemble those of Group I in the behaviour of the *He* lines and of [K], but show, in addition, strong lines typical of class A2. The effect is that of a composite spectrum containing (for example) an A2 spectrum superposed on a B8 ; and this may actually be the true explanation in some cases, such as in *v* Sagittarii, which is classified at Harvard as B8*p*, F2*p*. A special study of this star by J. S. Plaskett,* however, suggests that only one stellar atmosphere is responsible for the whole of this complex spectrum. Two other outstanding examples of this group are 88 Herculis and 4 Lacertæ. A complete list is given in Table II.

Group III : [*K*] *line weak, other A-type lines strong.*—In this group the stars should again be classified as B8 or B9 according to the *He* lines, but other lines, such as those of Fe^- at λ 4174, λ 4179, λ 4233, and λ 4385, the Si^+ double at λ 4128-31, the Sr^+ line at λ 4078, as well as other fainter lines typical of class A stars, are as strong as in types Ao to A2. In this case, however, the [K] line is comparatively faint, or even absent. This

* *Pub. Dom. Ap. Obs.*, IV, No. 1, 1927.

weakness of the [K] line may sometimes be merely a result of the falling off in the density of the spectrum in this region, especially in the poorer negatives, and such stars should be transferred to Group II ; but usually the negatives are sufficiently good to leave little doubt on this point. There is, in addition to the weak [K], another characteristic of these stars which serves to some extent to differentiate them from those of Group II. In many cases (though not in all) they are marked by the abnormal intensity of the Si^+ double, λ 4128–31, and of Sr^+ λ 4078, these lines being strong even

TABLE II

(*A-type lines strong*)

H.D.	Name	1900 R.A.	1900 Dec.	*m*	Sp.
		h m	° ′		
21291	HR 1035	3 21·0	+59 36	4·42	B9*p*
27295	53 Tau	4 13·5	+20 54	5·39	B8
145389	φ Her	16 5·6	+45 12	4·26	B9*p*
158261	HR 6506	17 23·2	+34 47	5·91	B9
161695	HR 6619	17 41·9	+31 33	6·25	B9
162732	88 Her	17 47·4	+48 25	6·43	B8
181615–6	v Sgr	19 16·0	−16 8	4·58	B8*p*, F2*p*
212593	4 Lac	22 20·4	+48 58	4·64	B8*p*
225289	HR 9110	23 59·9	+60 46	5·87	B9

Notes to Table II

21291. Very sharp lines ; Si^+ double also strong. H.D.C. remarks : " The lines are narrow."

145389. H.D.C. remarks : " The spectrum is peculiar in having the line K nearly as strong as in α CMa while the *He* lines 4026·3 and 4471·6 are also strong."

181615–6. H.D.C. remarks : " The spectrum is composite and variable. The line $H\beta$ is sometimes dark and sometimes bright. The lines of both components are narrow. A spectroscopic binary of class A." See also J. S. Plaskett (*loc. cit.*).

212593. Very sharp lines ; Si^+ double also rather strong. H.D.C. remarks : " The lines are narrow and the spectrum resembles that of β Orionis."

in comparison with the other A-type lines. Two good examples of this are H.D. 111133 and H.D. 164429. In some stars only the 4128–31 double shows this additional strengthening, and in two cases (H.D. 219749 and 67 *Oph*) the strength of this pair is the only peculiarity noted, other A-type lines being weak or absent. The complete list is given in Table III.

Group IV : Ionised oxygen lines strong for type.—The O^+ lines increase rapidly in intensity from types B3 to B0, being rarely visible in a normal B3 star and comparatively strong in B0 or B1. The spectral type can be determined independently from the lines $\lambda\lambda$ 4649 (C^{++}), 4089 and 4116 (Si^{+++}), and from the *He* and *H* lines. In many cases the O^+ appears to be abnormally strong for the spectral type as determined in this way, the principal lines affected being the triplet near λ 4070, and the lines at $\lambda\lambda$ 4350, 4367, 4415. A list of such stars is given in Table IV. In the case of stars

TABLE III

(*A-type lines strong but [K] weak*)

H.D.	Name	1900 R.A. h m	1900 Dec. ° ′	*m*	Sp.
22316	HR 1094	3 30·5	+56 36	6·27	B9
32549	11 Ori	4 58·9	+15 16	4·65	B9
58661	HR 2844	7 21·5	+48 23	5·57	B9*p*
78316	κ Cnc	9 2·4	+11 4	5·14	B8
79158	36 Lyn	9 7·3	+43 38	5·30	B8
90044	25 Sex	10 18·4	− 3 34	6·10	B9
111133	HR 4854	12 42·0	+ 6 30	6·39	B9
164429	HR 6718	17 56·0	+45 28	6·22	B9
170973	HR 6958	18 27·1	+ 3 35	6·34	B9
171247	HR 6967	18 28·6	+ 8 12	6·26	B8
182308	HR 7361	19 19·0	+64 13	6·33	B9
185872	14 Cyg	19 36·2	+42 35	5·39	B8
196178	HR 7870	20 30·6	+46 21	5·59	B9
201834	HR 8106	21 7·1	+53 9	5·73	B9
219749	HR 8861	23 13·2	+44 57	6·32	B9*p*

Notes to Table III

58661. H.D.C. remarks : " Several lines appear to be of increased intensity, especially a line near *Hε*, probably 3982·9."

78316. H.D.C. remarks : " A spectroscopic binary in which both components are bright."

201834. Very sharp lines.

219749. H.D.C. remarks : " The lines 4128·1 and 4131·1 are the strongest except those of hydrogen."

TABLE IV

(*O*$^{+}$ *lines strong for type*)

H.D.	Name	1900 R.A. h m	1900 Dec. ° ′	*m*	Sp.
14818	10 Per	2 18·2	+56 10	6·24	B2
31237	8 Ori	4 49·0	+ 2 17	3·87	B3
40111	139 Tau	5 51·8	+25 57	4·90	B2
41117	62 Ori	5 58·0	+20 8	4·71	B2*p*
149881	+14° 3086	16 32·4	+14 40	6·59	B2
185507	σ Aql	19 34·3	+ 5 10	5·17	B3
191877	HR 7716	20 7·0	+21 35	6·11	B0
201819	HR 8105	21 7·0	+35 53	6·40	B1
213420	6 Lac	22 26·1	+42 36	4·54	B3
217050	HR 8731	22 52·7	+48 9	5·20	B3
218376	1 Cas	23 2·4	+58 53	4·93	B1

Notes to Table IV

14818. Very sharp lines. H.D.C. remarks : " The lines are probably narrow. The line K is strong for this class."

31237. Very sharp lines.

41117. H.D.C. remarks : " The lines are narrow."

213420. Very sharp lines.

of type B3 the O^+ lines are of course not so strong as in the B1 to B2 stars, but they are strong enough to be easily visible and much stronger than is usual in stars of this type.

Group V: Inconsistent classifications.—This group contains stars for which the classification determined at the Norman Lockyer Observatory differs appreciably from that given in the Henry Draper Catalogue. Numerous small differences occur, such as changes from B3 to B4 or B8 to B9, but

TABLE V

(*Inconsistent classification*)

H.D.	Name	1900 R.A.	1900 Dec.	*m*	Spectrum H.D.	Spectrum N.L.O.
		h m	° ′			
32249	ψ Eri	4 56·6	− 7 20	4·81	B8	B3
36371	χ Aur	5 26·2	+32 7	4·88	B1	B3
36512	υ Ori	5 27·1	− 7 23	4·64	B3	B1
40967	3 Mon	5 57·1	−10 36	4·97	B8	B3
114376	15 CVn	13 5·1	+39 4	6·22	B9	B6
142763	HR 5931	15 51·2	+18 55	6·22	B9	B6
156651	+31° 2993	17 13·7	+31 37	7·20	B5	B9
165174	...	17 59·6	+ 1 55	6·09	B3	B1
170973	HR 6958	18 27·1	+ 3 35	6·34	B9	A2
193536	HR 7777	20 15·6	+46 0	6·28	B1	B3
200120	59 Cyg	20 56·4	+47 8	4·86	B0*p*	B2
200575	+56° 2523	20 59·2	+56 40	6·70	B8	B4
208095	HR 8357	21 48·8	+55 20	5·54	B3	B6
208501	13 Cep	21 51·5	+56 8	6·01	B9*p*	B6
209288	HR 8397	21 57·1	+10 29	6·36	B8	B4
209481	14 Cep	21 58·7	+57 31	5·50	B0	B2
216057	HR 8682	22 44·6	+53 53	6·08	B8	B4
217062	+59° 2607	22 52·8	+59 25	7·07	B9	B6
219634	HR 8854	23 12·2	+61 25	6·49	B8	B3

Notes to Table V

200120. H.D.C. remarks: "The lines $H\beta$, $H\delta$, $H\varepsilon$, $H\zeta$ are bright, and superposed on dark bands, which appear to be strongest on the side of greater wave-length."

208501. See notes to Table I.

are regarded as due to unimportant differences of judgment or lack of definition in the plates, and are not recorded here. Even larger differences, such as B3 to B5 or B5 to B8, which are frequently found and which are quite definitely appreciable, but for which no intermediate sub-type is available in the Harvard system, are not included. Only such differences as represent a change of more than one step in the Harvard sequence are mentioned. There appear to be three possible causes for these differences in classification:

(1) An observational error in the classification.
(2) Some peculiarity in the spectrum.
(3) An actual change in the spectral type.

Whichever may prove to be the true explanation of the inconsistencies, it was thought that a useful purpose might be served in drawing attention to such stars by their inclusion in the present note. An outstanding example is that of the star ψ Eridani, which is classified as B8 at Harvard, but in which the *He* lines are nearly as strong as in an average B2 star, and which has been classified at the Norman Lockyer Observatory (from a consideration of all the lines) as B3. This is a bright star (magnitude 4·8) with well-defined lines, and it would seem to be unlikely that a mistake could be made. Several other stars have been noted in which the difference of classification is as great as this. The complete list is given in Table V.

STELLAR HYDROGEN LINES AND THEIR VARIATION WITH TEMPERATURE AND SURFACE GRAVITY.

E. G. Williams, M.A.

(Communicated by the Director of the Solar Physics Observatory)

This investigation, full details of which will be published in the *Annals of the Solar Physics Observatory*, attempts to determine the variation in character of the hydrogen lines $H\beta$, $H\gamma$ and $H\delta$ with surface gravity and effective temperature. The results are based on 162 slit spectra of 82 stars of spectral types O to M. These were obtained with 2-prism spectrographs attached to the 25-inch Newall refractor and 15-inch Huggins refractor. The scale of the spectrograms at $H\gamma$ is 25 A. and 28 A. per mm. respectively. The plates were standardized with wedge spectra impressed closely adjacent to the star spectra. Stepped wedges of known transmission were fixed over the slits and uniformly illuminated by artificial sources. On the Huggins spectrograph a second slit was provided parallel to the first, enabling standardization to proceed during the star exposure.

Measurement of the instrumental redistribution in apparent wave-length of approximately monochromatic light showed that the spectrographs were capable of giving reliable contours for all except the narrowest hydrogen lines. A special developing tank was used to eliminate Eberhard effect. The contours have been obtained from the spectra with the aid of the photo-electric self-recording microphotometer of the observatory. The greatest two sources of error were uncertainty in drawing on the tracing the hypothetical continuous background over the line, and the coarse grain of the high-speed plates used.

The stars observed were those for which some measurement of effective temperature, T_e, has been made. The measurements of different observers were reduced to the Greenwich temperature scale. The surface gravities were computed by the usual formula, which assumes that the stars radiate as black bodies, the mass-luminosity curve having been used to get the masses.

After obtaining from the contours the total absorption for each line

observed, the following mean absorption ratios were found for the different spectrum ranges by weighting each pair of lines observed in any one spectrum according to their quality.

	O–B5	B8–A	F	G–M	All Stars
$H\beta : H\gamma$	0·95	0·84	0·91	0·74	0·85
$H\delta : H\gamma$	1·17	1·00	1·25	0·79	1·06

The unit of absorption employed is complete absorption over one unit of wave-number. The method adopted in drawing in the continuous spectrum for each type and line will have a marked influence on these figures. No variation of the ratios with absolute magnitude was detected. $H\gamma$ was observed in all stars, and $H\beta$ and $H\delta$ in 56 and 52 respectively. The above ratios enabled $H\beta$ and $H\delta$ to be used to obtain a mean absorption for $H\gamma$ of increased weight for stars in which those lines were observed. A plot of this measure of hydrogen absorption against T_e shows that the maximum occurs for dwarf stars at a temperature between 10,500° and 12,500° (types A5 and A0). For these stars the maximum is pronounced. The curve best representing the points for the 30 dwarf stars ($\log g > 4{\cdot}1$, g in dynes/cm.^{2}) gives the following figures :—

T_e	6,000°	7,000°	9,000°	11,000°	14,000°	20,000°
H absorption	90	170	470	880 (max.)	500	210

At the maximum there is a large negative absolute magnitude effect such that a star with surface gravity 20 times that of another has about 5 times as much hydrogen absorption. The absolute magnitude effect vanishes at type F and becomes positive for later types. These observations do not harmonize with the theoretical investigations of Milne and Pannekoek, which require the positive absolute magnitude effect of low temperatures to persist through the hydrogen maximum. It appears that inter-molecular Stark effect, which has not been taken into account by these investigators and which may increase the total absorption in a line, is insufficient to reconcile theory and observation.

The hydrogen intensity in α Orionis (Ma) is found to be greater than that of normal stars of type G. A pronounced statistical correlation between T_e and line depth has been discovered. The mean central intensity for B-type stars is 53 per cent. of the continuous spectrum, for A stars 35 per cent., and for F, G and K stars about 27 per cent. The mean for all is 36 per cent. This again is in disagreement with theory, which demands considerably lower residual intensities. For the late type stars there is evidence that the giants have deeper hydrogen lines than the dwarfs. For the hot stars no such relation exists, but here the effect of axial rotation, which broadens a line at the expense of its depth, is detectable. A plot of departure of depth from mean for type against Elvey's value of rotational velocity derived from the contour of λ 4481 (Mg^+), for 14 stars, indicates a filling of the line centre by 12 per cent. per 100 km./sec. of projected equatorial rotational velocity. The removal of this effect would reduce the central intensity in type B to 30 per cent.

In early type stars an exponential form of contour seems to represent the majority of hydrogen lines. In late types the lines seem to be relatively broader near the centre and narrower in the wings, but blending and inadequate dispersion complicate matters.

INTENSITY DISTRIBUTION IN EMISSION BANDS OF NOVÆ.

J. Genard, D.Sc. (Plates 2, 3.)

Introduction

No definite interpretation of emission bands in novæ spectra has been given up to the present time. The great brightness of the radiations makes it difficult to find a satisfactory hypothesis of their origin. Moreover, the bright radiations are generally accompanied by absorption bands of the same chemical elements ; these bands are then considerably shifted towards the violet, and are usually very strong. A correct hypothesis should give a complete explanation of the following characteristics of these spectra : (1) the width of the emission bands ; (2) their great intensity ; (3) the presence of strong absorption bands shifted towards the violet.

The best-known hypothesis is the one developed by Beals,* according to which the bright bands are due to a continuous ejection of atoms from the star by radiation-pressure. The star is thus surrounded by a very wide expanding envelope of gaseous material. To explain the abnormal widening of the bands, one must assume a very high velocity of ejection, of the order of about 1500 to 2000 km./sec. The absorption bands shifted towards the violet are due to the absorption of the continuous spectrum of the star by the external layers crossed by the line of sight, these layers having the highest radial velocity.

Beals's assumptions are : (1) radiation from each atom ejected contributes equally to the intensity of the emission band ; (2) the ejection velocities are equal or distributed according to the error law.

According to these assumptions, the band contours must be flat-topped, the width depending principally upon the minimum velocity of ejection.

Beals's explanation holds for novæ and Wolf-Rayet spectra. The curves of photographic density as a function of radial velocity, given in his last paper, for hydrogen lines in the spectra of Nova Aquilæ and Nova Cygni, seem to agree with the hypothesis.

Discussion

If we assume the radiation-pressure to be the only—or, at least, the most prominent—cause of the ejection of atoms which produce the bright bands in novæ spectra, Beals's assumptions are undoubtedly correct. Hence for

* Beals, *M.N.*, **90**, 202, 1929 ; *Pub. D.A.O.*, **4**, No. 17, 1930 ; *M.N.*, **91**, 966, 1931.

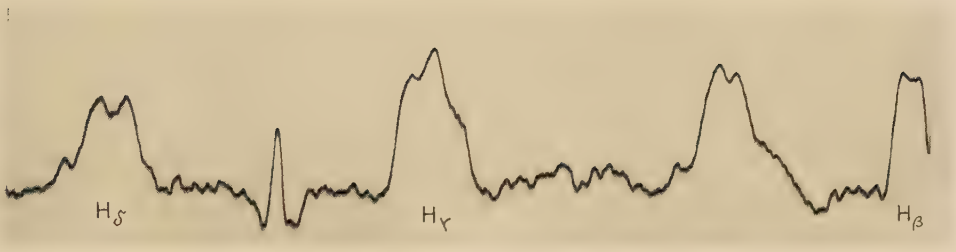


Fig. 1.—Nova Lacertæ, 1911 January 29.

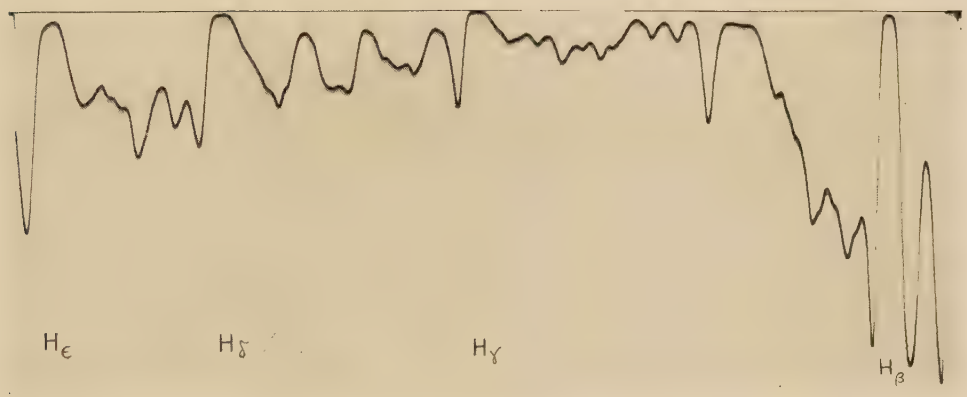


Fig. 2.—Nova Cygni, 1920 September 9.

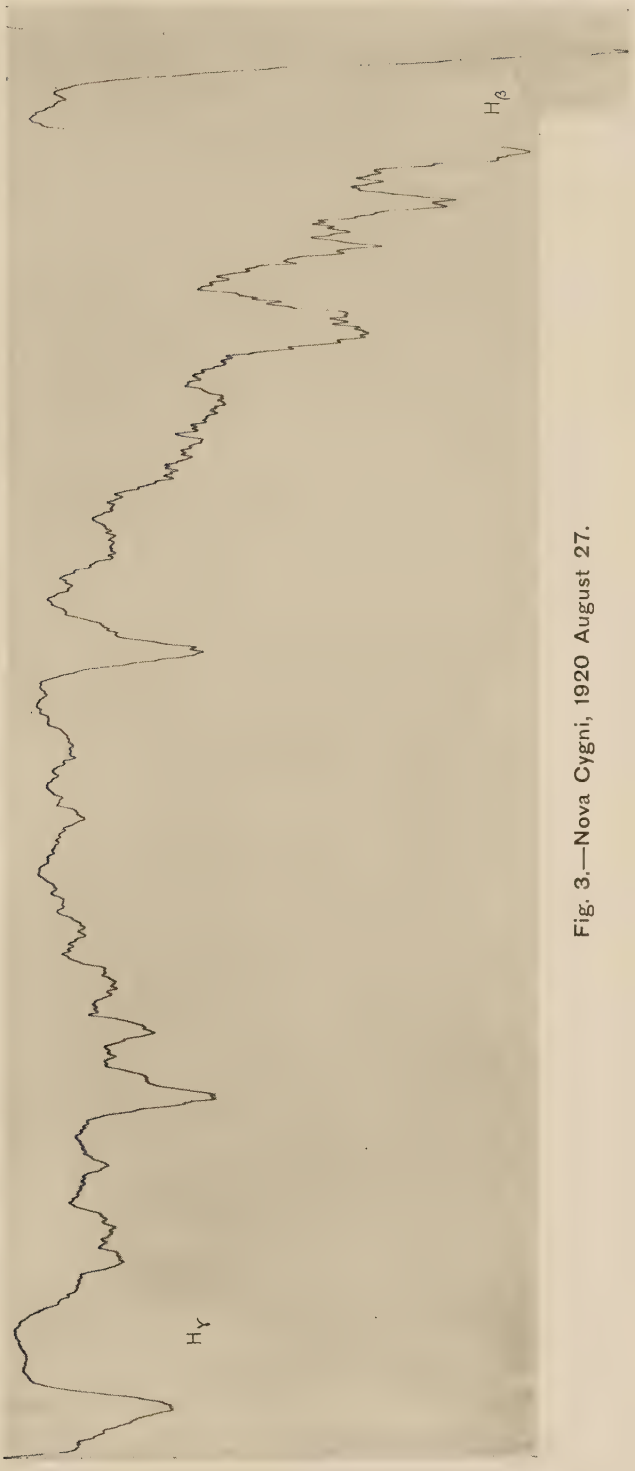


Fig. 3.—Nova Cygni, 1920 August 27.

the observational facts to agree with the theoretical results, the microphotometric records of bright bands must, in general, be flat-topped. Beals's theory could hardly explain important variations of intensity within those bands. We do not agree with his opinion that complex bands might also appear but would not be a serious objection to his theory.

To test this point, the best photographic and photometric conditions available are undoubtedly required. Thus, if any variation of intensity within the bands is to be detected, the spectrographic plates have to meet some definite requirements as far as dispersion and intensity are concerned. We think it impossible to detect such variation of intensity if the following conditions are not complied with : (1) for an average spectral dispersion, the plates must be slightly under exposed ; in fact, it is impossible to record small variations with strong or even ordinary spectra ; (2) if stronger plates are required for any reason, a larger dispersion must be used.

As examples, we give (Plates 2, 3, figs. 1, 2 and 3) reproductions of microphotometric records of the following spectra :—

Fig. 1.—Nova Lacertæ, 1911 January 29, medium dispersion (40 A./mm.), lightly exposed plate.
,, 2.—Nova Cygni, 1920 September 9, medium dispersion (40 A./mm.), strong plate.
,, 3.—Nova Cygni, 1920 August 27, wide dispersion (25 A./mm.), normally exposed plate.

Observational Data

A collection including about ten spectra of Nova Lacertæ and Nova Cygni was used for this study ; the plates were taken by MM. Idrac and Burson at the Meudon astrophysical observatory of the University of Paris. Unfortunately, these plates had not been standardised. It was thus impossible to measure the absolute intensities, but for purely qualitative work it is sufficient to determine the photographic densities. The microphotometric records, taken with the preceding precautions, and especially those reproduced in figs. 1 and 3, show that the bands are not flat-topped. Fig. 2, on the contrary, shows flat-topped curves similar to those given by Beals.

From records 1 and 3 we have plotted the density as a function of radial velocity for $H\beta$, $\lambda 4658$ (nebulium) and $H\gamma$ radiations. These bands are free from blending on the plates ; $H\gamma$ of Nova Cygni only is slightly blended with $\lambda 4364$ (nebulium) ; we have therefore excluded from the measurements the part of $H\gamma$ towards the larger wave-lengths.

Figs. 4, 5, 6, 7, 8 and 9 correspond respectively to

Fig. 4.—Nova Lacertæ, 1911 January 29, $H\beta$.
,, 5.— ,, ,, $\lambda 4658$ (nebulium).
,, 6.— ,, ,, $H\gamma$.
,, 7.—Nova Cygni, 1920 August 27, $H\beta$.
,, 8.— ,, ,, $\lambda 4658$ (nebulium).
,, 9.— ,, ,, $H\gamma$.

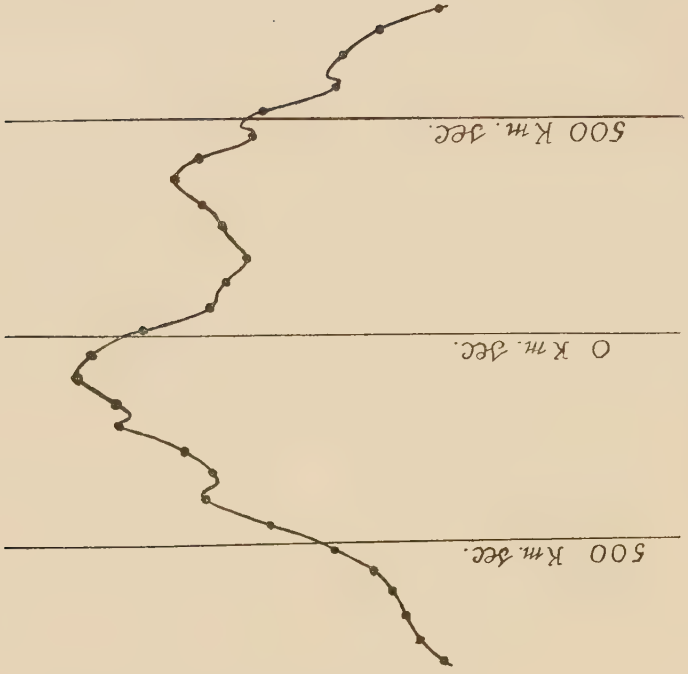


FIG. 5.—*Nova Lacertæ*, 1911 *January* 29, λ 4658 (*nebulium*).

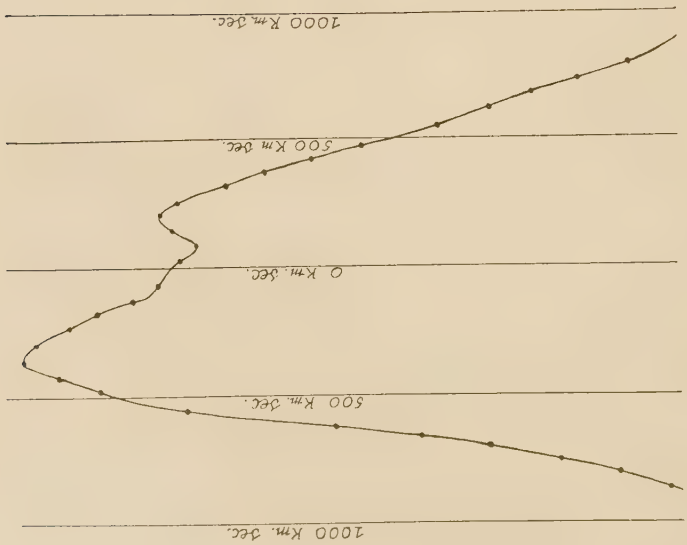


FIG. 4.—*Nova Lacertæ*, 1911 *January* 29, Hβ.

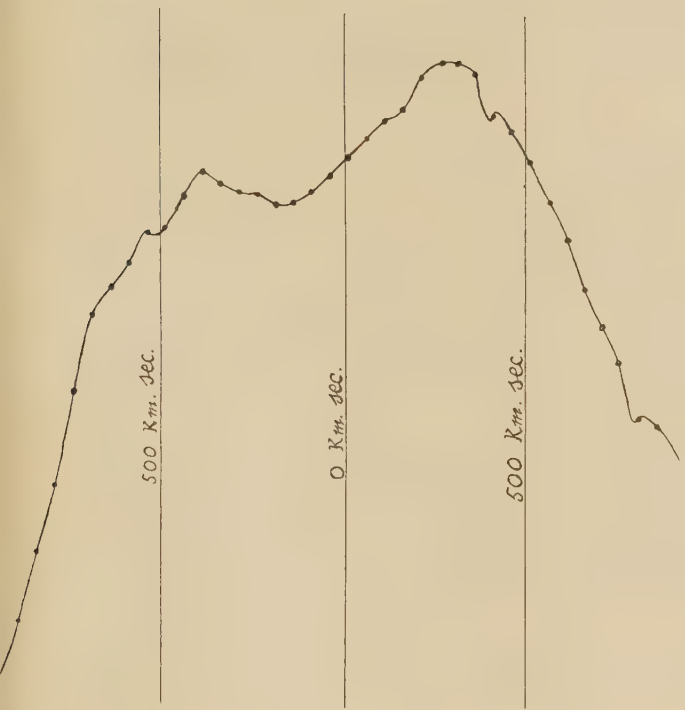


FIG. 6.—*Nova Lacertæ*, 1911 *January* 29, Hγ.

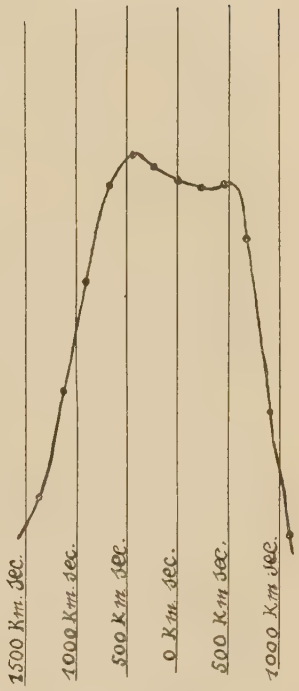


FIG. 7.—*Nova Cygni*, 1920 *August* 27, Hβ.

FIG. 8.—*Nova Cygni*, 1920 *August* 27, λ 4658 (*nebulium*).

FIG. 9.—*Nova Cygni*, 1920 *August* 27, Hγ.

The density scale used in these plots is arbitrary, because it was impossible to record the maximum of light on account of the high sensitivity of the microphotometer. This does not alter the qualitative significance of the curves.

Discussion of Results

As a result of the above diagrams, we cannot accept entirely Beals's theory of the formation of bright bands in novæ spectra. The continuous ejection of atoms under the effect of radiation-pressure does certainly exist at the surface of temporary stars, but another, discontinuous and irregular, phenomenon probably coexists with the continuous one. Moreover, we believe that this discontinuous phenomenon is of greater importance than the ejection of atoms by radiation-pressure.

The irregularity of the ejection of gaseous material from the star is proved by the fact that the points of maximum intensity move rapidly within the bands. As an example, two successive spectra of Nova Cygni taken one hour apart show sometimes a very appreciable movement of the maxima.

It does not appear possible to develop any definite and complete theory of the atomic ejection. The primary phenomenon appears to be of a "volcanic" nature, violent and more or less temporary streams of gaseous material being projected from the surface of the star. Explosions seem to break out frequently at random, and give rise to the maxima of intensity of the spectra.

The novæ seem not to be rather stable stars, ejecting gaseous material continuously in every direction under the effect of radiation-pressure alone. On the contrary, they appear to be undergoing a complete physical transformation.

Summary

Rather important variations of intensity have been observed within novæ bands.

Beals's theory of continuous ejection of atoms under the effect of radiation-pressure does not seem to agree perfectly with the observational facts, according to which the phenomenon of ejection appears as very irregular and similar to volcanic explosions.

The preceding records have been taken with the microphotometer of the Physics Department of the University of Paris. I am indebted to M. Challonge for the use of this apparatus.

The collection of plates has been placed at my disposal by M. Burson ; I wish to thank him very much.

Astrophysical Observatory of the
University of Paris, Meudon :
1931 *December* 20.

NOTE ON H. H. PLASKETT'S PAPER, "THE FORMATION OF THE MAGNESIUM *b* LINES IN THE SOLAR ATMOSPHERE."

G. G. Wiles.

(Communicated by Prof. E. A. Milne)

1. *Introduction.*—In his paper, "The Formation of the Magnesium *b* lines in the Solar Atmosphere," * Professor Plaskett has described his observations of certain line contours in the solar spectrum. He measured these at three different points on the Sun's disc. The results were applied to three idealised stellar atmospheres—one of pure absorption, where the light absorbed from a line is converted into heat in the atoms of the atmosphere ; one of pure scattering, where the light from the line is merely scattered by the atoms in random directions without change of wave-length ; and one of combined absorption and scattering, where both these effects are acting together.

Using the contour observed at the centre of the Sun's disc, Plaskett found that the theory of the simple models failed to give a satisfactory prediction of the contours nearer the edge of the disc. But using two contours, he found that a good prediction of the third was given by the model of combined absorption and scattering. The general conclusion was that this last-named model satisfactorily represented the observations. This conclusion led to several consequences. One was that the lifetime of an atom in the excited state increases with wave-length from the line wings in towards the line centre.

The present note concerns only the model of combined absorption and scattering. It is shown on physical grounds that a modification is necessary in the method of comparing the theory with the observations. The model is still found to represent the observations satisfactorily ; but the conclusion that the lifetime of an excited atom increases towards the line centre is found to be invalid.

2. *Statement of the Problem.*—Let r denote the ratio of the intensity of radiation at a given wave-length within the line to the intensity in the continuous spectrum just outside the line. Plaskett has shown that in the model of combined absorption and scattering, r is given by an equation (eqn. 34) of the form

$$r(\theta) = \phi(\theta, n, \xi), \tag{1}$$

where ϕ is a known function, θ is the angle between the direction of the radiation and the outward drawn normal to the surface of the star, and n and ξ are defined by

$$n = (\kappa + s)/\kappa_0, \qquad \xi^2 = \kappa/(\kappa + s). \tag{2}$$

* *M.N.*, **91**, 870, 1931.

s is the coefficient of scattering, and varies with wave-length through the line. It represents the absorption by atoms which subsequently re-emit quanta of the same wave-length in random directions. κ is the coefficient of pure absorption. κ_0 is the coefficient of general absorption just outside the line. But the physical mechanism responsible for κ_0 acts on radiation of all wave-lengths both inside and outside the line. Hence inside the line κ may be regarded as the sum of two parts,

$$\kappa = \kappa_0 + \kappa_i, \tag{3}$$

where κ_0 varies only inappreciably through the line, while κ_i varies appreciably. κ_i represents the effect of those excited atoms which become de-excited by hyperelastic collisions.

The problem is to find whether equations of the type (1) are consistent with the observed contours.

Each of the contours is associated with a definite value of θ. Plaskett has shown that n and ξ are not independent of θ. For they represent average values occurring at some average depth in the atmosphere. But at different values of θ we see to different average depths. There seems no way out of the difficulty, so that n and ξ are nevertheless assumed independent of θ.

Let the three observed contours be represented by r_1, r_2 and r_3. Then at each wave-length we have three equations of the type (1), viz.

$$r_1 = \phi_1(n, \xi), \tag{4}$$

$$r_2 = \phi_2(n, \xi), \tag{5}$$

$$r_3 = \phi_3(n, \xi), \tag{6}$$

where ϕ_1, ϕ_2 and ϕ_3 are known functions. If at each wave-length there can be found physically possible values of n and ξ satisfying these equations simultaneously, then the model of combined absorption and scattering is consistent with observation.

3. *Region of Physically Possible Values of* ξ.—The conditions for physically possible values of n and ξ may be stated concisely by the following three inequalities, at once obvious from equations (2) and (3),

$$1 > \xi^2 > 1/n > 0.$$

In fig. 1 these conditions are shown graphically for the line λ 5184. With every point (λ, ξ) in the diagram there is associated a value of n. This is such that if n and ξ are substituted in equation (4), they give the value of r_1 actually observed at the wave-length λ. The line AB represents $\xi = 1$, and points above this violate the condition $1 > \xi^2$. The curve CD represents $\xi^2 = 1/n$, and points below this violate the condition $\xi^2 > 1/n$. The curve BE represents $1/n = 0$, and points within this violate the condition $1/n > 0$. There remains the area $ABCDA$, and only in this are the values of n and ξ physically possible.

It is interesting to note that a further physical consideration now limits the curves for ξ which are possible even within the region $ABCDA$. $\kappa_i + s$ represents the capacity of the material to obstruct the flow of radiation over and above the obstruction due to general absorption. Hence $\kappa_i + s$ increases

with wave-length from the line wing towards the line centre. But from equations (2) and (3) we find

$$(\kappa_i + s)/\kappa_0 = n - 1.$$

Therefore $1/n$ must decrease from wing to centre. The light continuous curves in fig. 1 represent the equations

$$1/n = a,$$

where a has some positive constant value for each curve. Hence a physically possible curve for ξ must cross these progressively from wing to centre.

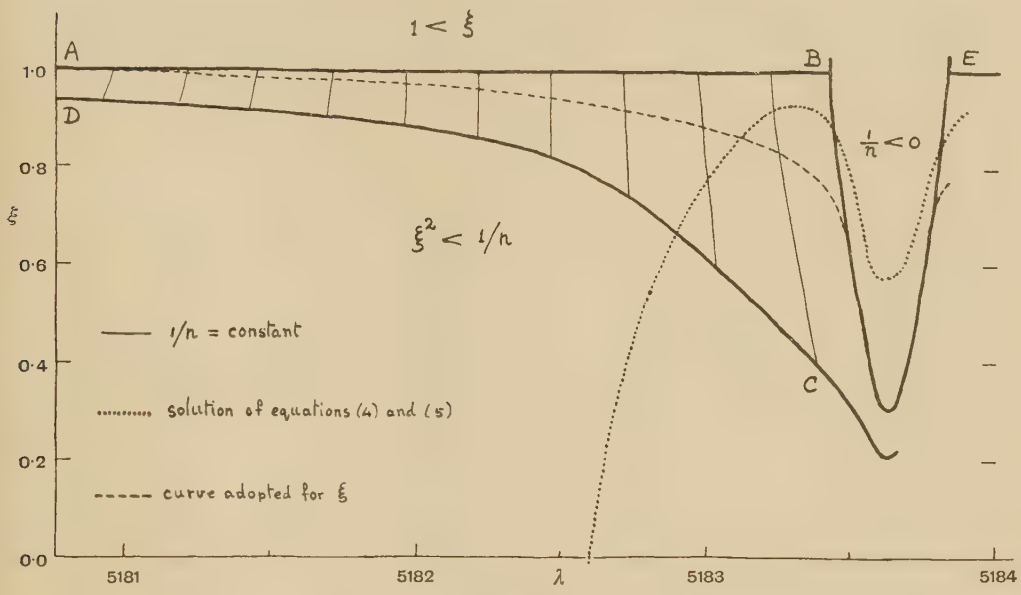


FIG. 1.

It is unfortunate that we cannot assume either κ_i or s to increase separately towards the line centre, for we could then impose a more limiting restriction on the possible curves for ξ. Actually κ_i and s do increase separately towards the centre. But that is not a foregone conclusion, but one of the results of the investigation.

4. *Former Method of comparing Theory with Observation.*—The problem now is to find those physically possible values of n and ξ which give best agreement with the observed contours. The procedure adopted by Plaskett was to solve equations (4) and (5) for n and ξ. These values were then substituted in (6). This gave a prediction of r_3, to be compared with observation. The method was, however, modified at certain wave-lengths, where negative roots were obtained for $1/n$ or ξ. In such cases the value zero was adopted instead of the negative value, and substituted in (4). This was solved for the other exponent. These new values were then substituted in (5) and (6) to give predictions of r_2 and r_3. The predictions found in this way are shown as closed circles in fig. 2.

The values of ξ found by solving (4) and (5) at various wave-lengths are indicated by the dotted curve in fig. 1. A glance at this shows that even when the negative values of $1/n$ and ξ are changed to zero, the curve still lies mostly outside the region of physically possible values. In the wings there is also another solution not shown in the figure. It exceeds unity, and so lies entirely outside the region of physically possible values.

It appears from this that the procedure of solving (4) and (5) is not successful. For we set out to find values of n and ξ in good agreement with the observed contours, and find for the most part only physically impossible values.

5. *Present Method of comparing Theory with Observation.*—A different procedure is adopted here. A physically possible curve within the area *ABCDA* is chosen arbitrarily. Values of n corresponding with these values

Table I

λ 5184

λ	ξ	$1/n$	r_2	r_3	λ	ξ	$1/n$	r_2	r_3
5180·80	1·00	0·89	0·951	0·964	5183·62	0·31	0·00	0·250	0·275
81·00	1·00	·86	·940	·955	83·68	·38	·00	·293	·327
81·60	0·98	·83	·925	·945	83·76	·70	·00	·452	·527
82·20	·96	·75	·889	·918	83·80	·75	·02	·483	·564
82·40	·95	·69	·862	·898	83·88	·80	·08	·532	·615
82·60	·93	·61	·822	·866	83·96	·83	·14	·572	·654
82·80	·91	·49	·766	·821	84·82	·96	·69	·860	·896
83·00	·88	·34	·688	·756	85·10	·98	·77	·901	·927
83·20	·84	·18	·599	·678	85·30	0·99	·82	·923	·943
83·32	·80	·10	·546	·627	85·50	1·00	·85	·935	·952
83·40	·76	·05	·505	·585	87·00	1·00	·96	·982	·987
83·48	·69	·00	·448	·522	87·20	1·00	·96	·984	·988
83·56	·43	·00	·324	·364	87·40	1·00	·99	·994	·996

of ξ are found from equation (4), using observed values of r_1. The values of ξ and n are then substituted in (5) and (6) to give a prediction of r_2 and r_3. Other arbitrary curves within *ABCDA* are then tried in the same way. Hence by a process of trial and error we find the curve for ξ which gives values of r_2 and r_3 in best agreement with the observed contours.

The details for the curve adopted for ξ for the line λ 5184 are given in Table I. The curve itself is shown as a dashed curve in fig. 1. The predicted contours are shown by crosses in fig. 2 along with the observed contours for comparison. There is seen to be satisfactory agreement.

It should be stated that the final curve for ξ cannot be chosen with great precision. For if the fit of one part of a contour be improved, it may tend to spoil the fit of other parts, as well as the fit of the other contour. In addition, a considerable change in the value of ξ makes only a small change in the predicted values of r_2 and r_3.

The conclusion is that the model of combined absorption and scattering gives a satisfactory explanation of the observed contours. But a knowledge of the observed contours within a reasonable degree of accuracy does not supply an accurate estimate of the variables n and ξ.

6. *Atomic Life as a Function of Wave-length.*—The ratios between the absorption and scattering coefficients are of physical interest. They are related to ξ and n through equations (2) and (3). We find three of these ratios given by

$$\frac{\kappa_i + s}{\kappa_0} = n - 1, \qquad \frac{\kappa_i}{\kappa_i + s} = \frac{\xi^2 - 1/n}{1 - 1/n}, \qquad \frac{s}{\kappa_i} = \frac{1 - \xi^2}{\xi^2 - 1/n}.$$

Their numerical values have been computed from the values of ξ and n in Table I. They are given in Table II at various distances $\Delta\lambda$ from the line

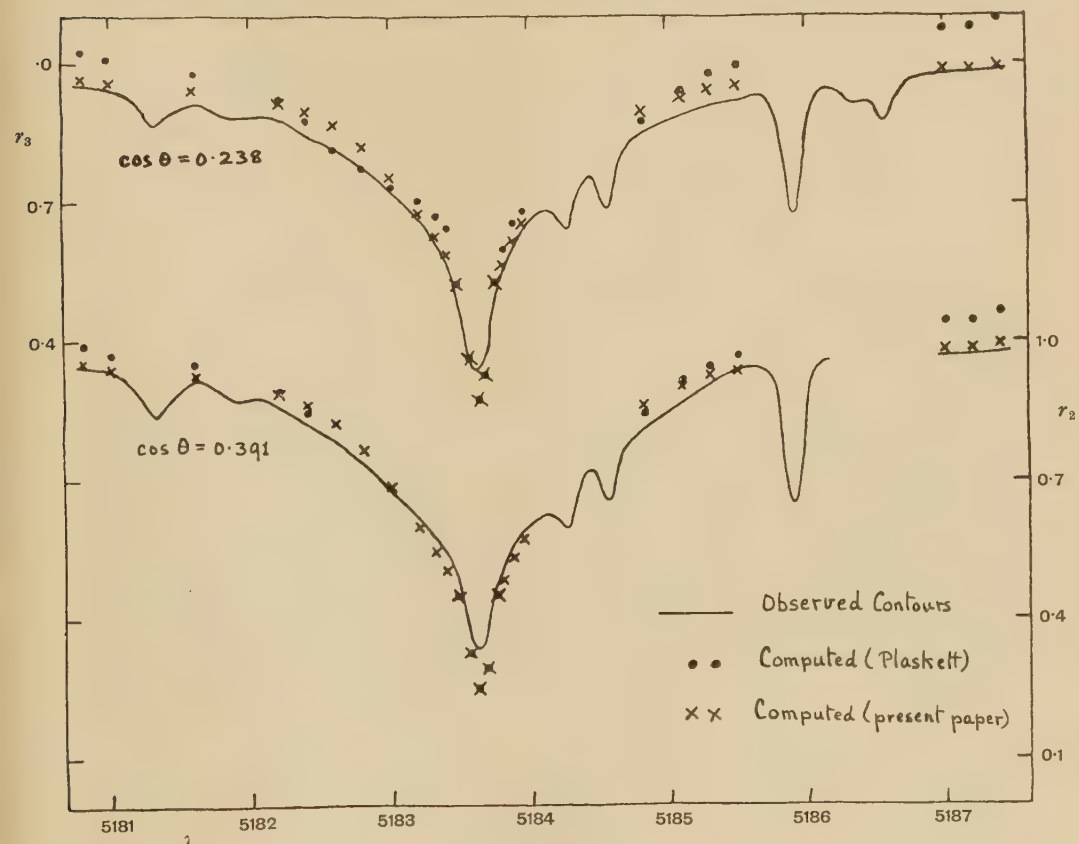


FIG. 2.

centre. The first ratio compares the obstruction to radiation by atoms producing the line, with the obstruction producing the general fogginess. The second is the fraction of excited atoms that become de-excited by hyperelastic collisions. The third shows the relative importance of the scattering and the pure line absorption. The scattering is seen to be important only very close to the line centre.

It must be repeated that the numerical values cannot be accepted too literally, owing to the limitation of the method mentioned in the previous section. But the numerical values do show the same variations through the line as is to be expected on physical grounds.

In the original paper these ratios were found to vary in an unexpected way. This was attributed to an increase from line wing to line centre of the average life of an atom in the excited state. But the argument is no longer valid.

TABLE II

λ 5184

$\Delta\lambda$	$\frac{\kappa_i+s}{\kappa_0}$	$\frac{\kappa_i}{\kappa_i+s}$	$\frac{s}{\kappa_i}$	$\Delta\lambda$	$\frac{\kappa_i+s}{\kappa_0}$	$\frac{\kappa_i}{\kappa_i+s}$	$\frac{s}{\kappa_i}$
−2·82	0·12	1·00	0	0·00	∞	0·10	9·3
−2·62	0·16	1·00	0	0·06	∞	0·14	5·9
−2·02	0·21	0·77	0·3	0·14	∞	0·49	1·04
−1·42	0·34	0·69	0·45	0·18	50	0·56	0·80
−1·22	0·45	0·68	0·48	0·26	12	0·61	0·64
−1·02	0·65	0·66	0·52	0·34	6·4	0·64	0·56
−0·82	1·04	0·66	0·51	1·20	0·46	0·75	0·33
−0·62	1·93	0·66	0·52	1·48	0·29	0·83	0·21
−0·42	4·5	0·64	0·56	1·68	0·21	0·89	0·1
−0·30	8·6	0·60	0·67	1·88	0·17	1·00	0
−0·22	18	0·55	0·81	3·38	0·04	1·00	0
−0·14	∞	0·48	1·10	3·58	0·04	1·00	0
−0·06	∞	0·18	4·4	3·78	0·01	1·00	0

At any point let $N(\lambda)d\lambda$ be the average number of atoms per cm.^{3} capable of emitting radiation of a wave-length between λ and $\lambda+d\lambda$. Then the number de-excited by collisions per second is $N(\lambda)d\lambda\psi(f)$, where f denotes the frequency of collisions of various types, and $\psi(f)$ is positive and increases with the frequencies. The number emitting λ to $\lambda+d\lambda$ radiation per second is approximately $a_{21}N(\lambda)d\lambda$, where a_{21} is the probability of a spontaneous emission (Einstein coefficient). Also let $n(\lambda)d\lambda$ be the average number per cm.^{3} which become capable of emitting λ to $\lambda+d\lambda$ radiation per second. Of these a fraction $\kappa_i/(\kappa_i+s)$ are subsequently de-excited by collisions, and a fraction $s/(\kappa_i+s)$ subsequently emit λ to $\lambda+d\lambda$ radiation. Hence we have

$$\frac{\kappa_i}{\kappa_i+s}n(\lambda)d\lambda=N(\lambda)d\lambda\psi(f), \qquad \frac{s}{\kappa_i+s}n(\lambda)d\lambda=a_{21}N(\lambda)d\lambda.$$

Hence

$$s/\kappa_i=a_{21}/\psi(f). \tag{7}$$

As we move from line wing to line centre we see to a smaller average depth in the atmosphere, and therefore to a smaller value of $\psi(f)$. Thus by (7) it is natural to expect s/κ_i to increase towards the centre, as it does in Table II (last column). We can therefore draw no conclusion concerning a variation of a_{21} or its reciprocal, the atomic life.

7. *Summary.*—In "The Formation of the Magnesium *b* lines in the Solar Atmosphere," Plaskett showed a model atmosphere of combined absorption and scattering to be consistent with observed line contours in the solar spectrum. But his method needed modification on physical grounds. In the present note a somewhat different procedure has been adopted. The model of combined absorption and scattering is still found to be satisfactory.

But the ratios of the various absorption and scattering coefficients are found to vary with wave-length through the line in a less unexpected way. Hence it is no longer possible to conclude that the atomic life of an excited atom varies with wave-length through a line.

In conclusion, it is a pleasure to record my thanks for helpful discussion and criticism to Professor E. A. Milne and to Professor H. H. Plaskett.

Balliol College, Oxford :
1932 *February* 27.

DIAMAGNETISM AND DRIFT CURRENTS IN THE SOLAR ATMOSPHERE.

T. G. Cowling, M.A., D.Phil.

(Communicated by Professor S. Chapman)

1. Among the problems of electromagnetic theory encountered in a study of solar phenomena are the problems of the maintenance and radial limitation of the general solar magnetic field and of the sunspot magnetic field. The more recent work relating to these phenomena has included papers by Professor S. Chapman * and Dr. Ross Gunn.† In an earlier paper by the present writer ‡ Chapman's work was discussed, and the effect on the magnetic field of the drift currents to which he ascribed the limitation of the general field was shown to be masked by another effect.§ In the same paper certain general considerations relating to Gunn's work were advanced. It is the object of the present paper to give a more detailed discussion of certain features of his work.

His work includes, *inter alia*, explanations of the radial limitation of the general field and of the maintenance of the spot field. Under the action of a magnetic field, electrons describe spiral paths about the lines of magnetic force. The magnetic effect of such a spiral motion is to render the gas diamagnetic, the intensity of diamagnetism being greatest when electrons can spiral freely between collisions, *i.e.* when they can spiral several times about the lines of force between successive encounters. Thus, he suggests, there will be a strongly diamagnetic layer in the solar atmosphere, through which the lines of magnetic force will be unable to pass : this will explain the outward limitation of the general field. Again, under the action of a non-homogeneous magnetic field, electrons possess, superposed on their spiral

* *M.N.*, **89**, 57, 1928. In this paper references to earlier work relating to the general field are given.

† *Phys. Rev.*, **32**, 133, 1928 ; **33**, 614, 832, 1929 ; **34**, 335, 1929 ; **35**, 635, 1930, etc. ; *Astrophys. Journ.*, **69**, 287, 1929.

‡ *M.N.*, **90**, 140, 1929.

§ Professor Chapman desires me to state that he accepts the conclusions of that paper, while believing that the explanation of the phenomenon must be somewhat similar to that which he advanced (but depending on the departure from thermodynamic equilibrium).

motion, a velocity of drift across the lines of force : he suggests that, in consequence, in a sunspot currents are set up which maintain the field.

The theory of diamagnetism in the solar atmosphere is similar to the theory, first advanced by Schrödinger,* explaining diamagnetism in metals. It has been shown that in metals the diamagnetic effect of spiralling may be completely masked by surface electric currents.† It seems possible that in an ionised gas the diamagnetic effect may similarly be masked. In the solar atmosphere, however, the thickness of the transition layer in which diamagnetism, according to Gunn, becomes important is many thousand times the radius of spiraling. He has therefore suggested that the effect cannot be compared with the diamagnetic effect in metals, in which the thickness in which diamagnetism becomes important is of the same order of magnitude as the spiral radius. The point, however, requires investigation, which will be given here.

2. We consider the problem of diamagnetism as it presents itself at the equator. It is sufficient to consider a gas in an isothermal steady state, as at no point does Gunn assume any departure from such a state ; and we shall neglect the curvature and rotation of the Sun. Thus, if Ox is taken as vertical and Oz as horizontal and in the S.-N. direction, the electric fields, of intensities E, H,‡ act parallel to Ox and Oz respectively, and the gas can be supposed stratified in plane layers parallel to $x=0$.

As was shown in the previous paper, there exists no resultant electric current in the gas under such conditions. Hence, if the diamagnetic effect is actually neutralised by electric currents, these currents must arise from the presence of a drift of electrons, which, like Chapman's drift, is masked by the spiralling which causes diamagnetism. Such a drift cannot be detected by considering the mean velocity, at a given instant, of all electrons near a given point : it is necessary, instead, to consider the mean motion of all electrons near a given point during the whole of their paths. For this purpose we employ the methods of the theory of diffusion. We consider the motion of a certain group of electrons, regarding them, for the moment, as particles of a type different from that to which the rest of the electrons belong. This group is chosen as follows : Let $x=x_1$ and $x=x_2$ be two levels such that $x_2 > x_1$. Then the group is chosen to include at a given instant a proportion λ of the electrons at any level, where λ is a continuous and differentiable function of x, such that

(i) $\lambda=1$ if $x_1 \leqslant x \leqslant x_2$;
(ii) λ decreases from 1 to 0 as x passes from x_1 to x_1-h or from x_2 to x_2+h, where h is small compared with x_2-x_1;
(iii) $\lambda=0$ elsewhere.

If h is sufficiently small, the mean velocity of electrons of this set is approximately equal to the mean drift of those electrons which at the given time lie between x_1 and x_2.

* *Wien Sitzungsberichte*, **121**, 1305, 1912.
† See van Leeuwen, *Jour. de Phys.*, **2**, 374, 1921.
‡ Electromagnetic units are used throughout.

3. The gas consists of particles of several different types. Let m_j, e_j denote the mass and charge of a particle of the jth type; let n_j denote the number of such particles per unit volume, and let the number per unit volume whose component velocities u_j, v_j, w_j lie in ranges du_j, dv_j, dw_j be $f_j du_j dv_j dw_j$. Then Boltzmann's equation for f_j is of the form

$$\frac{\partial f_j}{\partial t}+u\frac{\partial f_j}{\partial x}+\left(-g+\frac{e_j}{m_j}(E+v_jH)\right)\frac{\partial f_j}{\partial u_j}-\frac{e_ju_jH}{m_j}\frac{\partial f_j}{\partial v_j}$$
$$=-\sum_k\iiiint\!\!\iint(f_jf_k-f_j'f_k')g_{jk}bdbd\epsilon du_k dv_k dw_k \qquad (1)$$

(see Jeans, *Dynamical Theory of Gases*, p. 210; in this equation f_j', f_k' denote the results of substituting in f_j, f_k the velocities (u_j', v_j', w_j'), (u_k', v_k', w_k') after encounter of particles whose velocities before encounter were (u_j, v_j, w_j), (u_k, v_k, w_k), and the variables g_{jk}, b, ϵ are identical with the variables V, p, ϵ used by Jeans).

When no magnetic field is present, the solution of this equation is obtained as follows. Let us set

$$F_j=n_j(m_j/2\pi kT)^{3/2}\exp\{-m_j(u_j^2+v_j^2+w_j^2)/2kT\}. \qquad (2)$$

Then the solution is of the form

$$f_j=F_j(1+\chi_j) \qquad (3)$$

where χ_j is small. Therefore, on substitution from (3) into (1), we neglect χ_j on the left-hand side of the equation and the products $\chi_j\chi_k$, $\chi_j'\chi_k'$ on the right-side side. In obtaining the form of solution valid in the present case, the only difference in procedure is that those terms on the left-hand side of the equation in which χ_j appears multiplied by H are retained. This is necessitated by the fact that, when electrons are free to spiral about the lines of magnetic force, the effect of the magnetic field in deflecting the electrons is comparable with that of encounters.

The function F_j depends on t only by reason of the dependence of n_j on t; and n_j varies with t only because there is diffusion of particles of the j-th type from one level to another and because the rate of diffusion varies with the level considered. Thus $\partial n_j/\partial t$ is very small, and will be neglected. Also $F_j'F_k'=F_jF_k$. Hence, on substituting from (3) into (1), and making the approximations indicated, we obtain the equation

$$F_ju_j\left[\frac{1}{n_j}\frac{\partial n_j}{\partial x}-\frac{1}{kT}(-m_jg+eE)\right]+\frac{e_jHF_j}{m_j}\left(v_j\frac{\partial\chi_j}{\partial u_j}-u_j\frac{\partial\chi_j}{\partial v_j}\right)$$
$$=-\sum_k\iiiint\!\!\iint F_jF_k(\chi_j+\chi_k-\chi_j'-\chi_k')g_{jk}bdbd\epsilon du_k dv_k dw_k. \qquad (4)$$

If $H=0$, equation (4) reduces to the ordinary equation of diffusion. Its solution then is of the form

$$\chi_j=\alpha_ju_j$$

where α_j is a function of $u_j^2+v_j^2+w_j^2$. From this it follows that all the

diffusion is in the direction of Ox, as is to be expected. When, however, $H \neq 0$, the solution takes the form

$$\chi_j = \alpha_j u_j + \beta_j v_j \tag{5}$$

where α_j, β_j are both functions of $u_j^2 + v_j^2 + w_j^2$, which satisfy the equations

$$u_j\left[\frac{1}{n_j}\frac{\partial n_j}{\partial x} - \frac{1}{kT}(-m_j g + e_j E)\right] - \frac{e_j H u_j \beta_j}{m_j}$$
$$= -\sum_k \iiiint F_j(\alpha_j u_j + \alpha_k u_k - \alpha_j' u_j' - \alpha_k' u_k')g_{jk}\, bdbd\epsilon du_k dv_k dw_k\,; \tag{6}$$

$$\frac{e_j H \alpha_j v_j}{m_j} = -\sum_k \iiiint F_j(\beta_j v_j + \beta_k v_k - \beta_j' v_j' - \beta_k' v_k')g_{jk}\, bdbd\epsilon du_k dv_k dw_k. \tag{7}$$

From the form of (5) it follows that, in addition to diffusion parallel to Ox, there is now diffusion parallel to Oy, *i.e.* in a direction perpendicular both to the magnetic field and to the original direction of diffusion. This is in agreement with the experimental results as to the Hall effect.

4. Let us distinguish the group of electrons defined in § 2 by the suffix a. Then, if the total number of electrons per unit volume be denoted by n, $n_a = \lambda n$. Also the equation of equilibrium for all the electrons is

$$\frac{1}{n}\frac{\partial n}{\partial x} = \frac{1}{kT}(-m_a g + e_a E).$$

Thus

$$\frac{1}{n_j}\frac{\partial n_j}{\partial x} - \frac{1}{kT}(-m_a g + e_a E) = \frac{1}{\lambda}\frac{\partial \lambda}{\partial x} + \frac{1}{n}\frac{\partial n}{\partial x} - \frac{1}{kT}(-m_a g + e_a E) = \frac{1}{\lambda}\frac{\partial \lambda}{\partial x}. \tag{8}$$

Now, by the results of the previous paper, there is no diffusion of particles of any type other than the selected group of electrons and the remainder of the electrons. Let the component parallel to Oy of the velocity of diffusion of the selected set of electrons at any point be denoted by V. Then the number of these electrons crossing unit area of the plane $y = 0$ is $n_a V$. Also

$$n_a V = \iiint f_a v_a du_a dv_a dw_a$$
$$= \iiint F_a \chi_a v_a du_a dv_a dw_a$$
$$= \iiint F_a \beta_a v_a^2 du_a dv_a dw_a. \tag{9}$$

The values of $n_a V$ in the limiting cases of very large and very small density are found thus. By (2), F_j is proportional to n_j, and so, roughly speaking, to the density. Thus, by (7), α_j/β_j is proportional to the density. It follows that in (6) the ratio of the terms in α_j, β_j is proportional to the square of the density. Thus, when the density is small, equation (6) for the selected set of electrons approximates to

$$u_a\left[\frac{1}{n_a}\frac{\partial n_a}{\partial x} - \frac{1}{kT}(-m_a g + eE)\right] - \frac{e_a H u_a}{m_a}\beta_a = 0,$$

whence, by (8),

$$\beta_a = \frac{m_a}{e_a H} \cdot \frac{1}{\lambda} \frac{\partial \lambda}{\partial x}.$$

On using this value of β_a in (9), we have

$$n_a V = n_a \frac{kT}{e_a H} \cdot \frac{1}{\lambda} \frac{\partial \lambda}{\partial x} = \frac{nkT}{e_a H} \frac{\partial \lambda}{\partial x}. \tag{10}$$

Since, in obtaining this value of $n_a V$, the terms in (6) involving α_j, which depend on the effect of encounters, are neglected in comparison with those involving β_j, which depend on the effect of the magnetic field, this approximation corresponds to the case of free spiralling.

If, on the other hand, the density is large, so that the effect of spiralling between encounters is negligible, in (6) the term involving β_j is small compared with that involving α_j ; thus, by the above, α_j is inversely proportional to the density, and so, by (7), β_j is inversely proportional to the square of the density. Thus, by (9), $n_a V$ is inversely proportional to the density. Hence, if the density is very large, we can set $n_a V = 0$.

5. Consider now the number of electrons of the selected set which cross the plane $y = 0$ per unit extension in the z-direction in unit time. As the number crossing unit area of the plane per unit time at any point is $n_a V$, this number is equal to

$$\int_{x_1 - h}^{x_2 + h} n_a V dx.$$

Between the levels $x = x_1$ and $x = x_2$ the selected set includes all the electrons in the gas ; thus between these levels $V = 0$. The above number is thus equal to

$$\int_{x_1 - h}^{x_1} n_a V dx + \int_{x_2}^{x_2 + h} n_a V dx. \tag{11}$$

This expression can be used to derive known results. Suppose, for example, that at both of the levels x_1, x_2, electrons can spiral freely between encounters. Then in each of the integrals of (11) we can use (10) : thus, as h is small, (11) is approximately

$$\frac{kT}{e_a} \left\{ \frac{n_1}{H_1} \int_{x_1 - h}^{x_1} \frac{\partial \lambda}{\partial x} dx + \frac{n_2}{H_2} \int_{x_2}^{x_2 + h} \frac{\partial \lambda}{\partial x} dx \right\} = \frac{kT}{e_a} \left\{ \frac{n_1}{H_1} - \frac{n_2}{H_2} \right\},$$

where n_1, n_2, H_1, H_2 denote the values of n, H at the levels $x = x_1$ and $x = x_2$. If $x_2 - x_1$ is sufficiently small, this is approximately

$$\begin{aligned} & -(x_2 - x_1) \left[\frac{\partial}{\partial x} \left(\frac{nkT}{e_a H} \right) \right]_{x = x_1} \\ &= -(x_2 - x_1) \left[\frac{kT}{e_a H} \frac{\partial n}{\partial x} - \frac{nkT}{e_a H^2} \frac{\partial H}{\partial x} \right]_{x = x_1} \\ &= -n_1 (x_2 = x_1) \left[\frac{(-m_a g + e_a E)}{e_a H} - \frac{kT}{e_a H^2} \frac{\partial H}{\partial x} \right]_{x = x_1}. \end{aligned} \tag{12}$$

This is the number of electrons crossing an area approximately equal to $x_2 - x_1$ of the plane $y = 0$ per unit time. Thus the mean velocity of drift of electrons of the selected set parallel to Oy is made up of two parts, equal respectively to $(m_a g - e_a E)/e_a H$ and $(kT/e_a H^2)\partial H/\partial x$. Both of these drifts have already been studied ; the first is Chapman's drift current, and the second is that of Gunn.* Thus the results obtained by our method are consistent with those obtained by other methods.

6. Let us now consider the transition layer between the region in which electrons are free to spiral about the lines of magnetic force and the region in which they are not free to spiral. Let us suppose that at the level x_1 electrons are not free to spiral between encounters, but that at the level x_2 they are free so to do. Then in (11) we may set $n_a V = 0$ in the first integral, and use (10) in the second. The expression then becomes

$$\frac{kT}{e_a}\frac{n_2}{H_2}\int_{x_2}^{x_2+h}\frac{\partial\lambda}{\partial x}dx = -\frac{kT}{e_a}\frac{n_2}{H_2}. \tag{13}$$

Thus there is a drift current of electrons between the levels x_1 and x_2, whose direction is opposed to that of Chapman's drift ; the effect of this drift current is thus to oppose the radial limitation of the magnetic field. Since each electron carries a charge e_a, the passage of the number of electrons given by (13) across the plane $y = 0$ is equivalent to an electric current $-kTn_2/H_2$. Thus the motion of the selected group of electrons is equivalent to a plane sheet of electric currents parallel to $x = 0$, which flow in the direction of Oy and whose magnitude per unit extension in the z-direction is $-kTn_2/H_2$. The effect of such a sheet of currents is to tend to produce a difference $4\pi kTn_2/H_2$ in the magnetic fields above and below the sheet.

Now at the level $x = x_1$ the diamagnetic effect of spiralling is negligible, and at the level $x = x_2$ it produces an intensity of magnetisation $-n_2 kT/H_2$. Thus its effect is to tend to produce a difference $-4\pi n_2 kT/H_2$ in the magnetic fields at $x = x_2$, $x = x_1$. This is equal and opposite to the effect of the drift currents. The diamagnetic effect is thus completely masked by the effect of drift currents. Moreover, x_2 can have any value, subject only to the condition that electrons can spiral freely at the level $x = x_2$; it follows that the magnetic effect of drift currents exactly neutralises the effect of diamagnetism at all levels at which electrons can spiral freely. Thus Chapman's and Gunn's drift currents exactly neutralise the effect of the change of diamagnetic intensity with level at such levels. The results of the earlier paper are, in fact, equivalent to this result.

7. It appears, then, that in the transition layer in which electrons are becoming free to spiral under the action of the magnetic field there are drift currents similar to those which mask the diamagnetic effect of spiralling in a metal, and that these completely mask the diamagnetic effect in the gas. It also appears that, in the problem considered above, Gunn's second effect is masked by the diamagnetic effect. By similar arguments, it can be shown that the same masking occurs whenever a gas in an isothermal steady state

* Our result actually differs from Gunn's result by a constant factor : this is due to the admittedly approximate nature of his methods.

is considered. Thus it may be concluded that, since in Gunn's discussion of these effects no account is taken of departures from an isothermal steady state, neither his theory of the radial limitation of the Sun's general field nor his theory of the maintenance of the sunspot field can be accepted in its present form.

THE INFLUENCE OF A SOLAR ECLIPSE UPON UPPER ATMOSPHERIC IONIZATION.

S. Chapman.

1. In a recent lecture * I have suggested that the upper ionized layer in the Earth's atmosphere, at a height of about 220 km., is ionized by ultra-violet light, while the lower, at about 100 km. height, is ionized by neutral solar corpuscles. This hypothesis was proposed as the most likely one in the light of the available evidence, but it was not suggested that the arguments were conclusive. It is desirable to find other considerations enabling the hypothesis to be tested.

In January last Professor Appleton kindly communicated to me some passages bearing on this question from a provisional draft of a paper by himself and Mr. Naismith. They pointed out that during the solar eclipse of 1927 the radio experiments † showed a partial return to night conditions, a striking feature being that the equivalent height and the reflection coefficient of the ionized layer reached their maximum values at a time agreeing to within 1 second with that of the centre of totality. Adopting current estimates of the velocity of travel of the supposed neutral corpuscles from the Sun, they pointed out that "such a solar stream should take $4\frac{1}{2}$ minutes to travel from the Moon to the Earth. During a solar eclipse, therefore, we should expect the maximum wireless effect on the lower ionized region to show itself $4\frac{1}{2}$ minutes or more after totality. If there was any lag in the eclipse effect in 1927 it was certainly much less than this. We are therefore inclined to ascribe the ordinary diurnal variation of the ionization in the lower region to ultra-violet radiation, which travels the Moon's distance in just over 1 second."

2. On theoretical grounds I should expect a lag of at least some minutes in the maximum ion-reducing effect of the eclipse, even supposing an infinite speed of the ionizing agent from the Sun. This expected lag would be due to the continued recombination of the electrons, which for some minutes after the centre of totality would be likely to exceed the renewed electron-production. Hence the interpretation of the apparent absence of lag during the 1927 eclipse seemed difficult on either hypothesis as to the ionizing agent.

Further consideration, however, made it clear that, though in a different way, eclipse observations are admirably suited to settle the important question as to the source of the ionization of the lower layer. In discussing an

* Bakerian Lecture, *Proc. Roy. Soc.*, A, **132**, 353, 1931.

† E. V. Appleton, *Journ. Inst. Elec. Eng.*, **66**, 872, 1928; also *Radio Research Board Special Report*, No. 7, 1928.

eclipse on the corpuscular hypothesis many motions besides that of the particles radially outwards from the Sun must be taken into account, such as the solar rotation, the orbital velocities of the Earth and Moon, and the rotation of the Earth. It appeared that, of these, the motions of the Moon and Earth were of principal importance, and that, in consequence of them, the Moon should cut off the supposed neutral solar corpuscles more than an hour before the ultra-violet light is stopped ; the eclipse effect on the incidence of the corpuscles should in fact be entirely over before the optical eclipse begins. Thus on the corpuscular hypothesis the lower ionized layer should be subject to the influence of the eclipse long before the upper ionized layer is affected.

3. In a total solar eclipse the area of totality is always very small compared with the area of partial eclipse ; the normal section of the penumbral shadow cone, near the Earth, is approximately 4000 miles in diameter, while that of the cone of totality never exceeds 150 miles. The duration of totality is very short, 8 minutes or less ; while the whole duration of an eclipse may be as great as $3\frac{1}{4}$ hours at the equator near noon, or nearly 2 hours in high latitudes or near sunrise or sunset. Though there is a great difference between a total and nearly total eclipse so far as concerns spectacular effects or astronomical observations, the difference as regards their influence on the ionization of the upper atmosphere is slight. Perhaps in the past undue importance, from the standpoint of radio transmission, has been attached to the period of totality and the events happening during that period. No striking change in radio transmission marking out the few minutes of totality from the immediately adjacent periods of partial eclipse should be expected on either hypothesis as to the ionization. It seems improbable (§ 8) that the reduction in the ion-content of the layer ionized by ultra-violet light should be at its maximum during the period of totality ; my calculations suggest that the minimum ion-content in the layer thus ionized may not be attained until about 15 minutes after the centre of totality.

4. At present we have no observational proof that neutral corpuscles are continually emitted by the Sun, but E. A. Milne * has shown that they are likely to be emitted by radiation-pressure acting upon atoms that, through chance collisions in the chromosphere, gain an upward velocity of 50 km./sec. or more. S. R. Pike † has shown also that atoms which in like manner gain a moderately abnormal *horizontal* velocity component will also be vertically (as well as horizontally) accelerated, and (unless drawn back towards the Sun by neighbouring dark areas or sunspots) will leave the Sun altogether. Their calculations agree in predicting a limiting velocity of about 1600 km./sec., attained,‡ within about 6 per cent., by the time the atoms have reached a distance of 10 solar radii from the Sun.

By chance collisions an atom in the chromosphere may acquire an abnormal velocity in any direction, but from Pike's work it may be inferred that the sheaf of corpuscular rays thus emitted from any small solar area will be

* E. A. Milne, *M.N.*, **86**, 459, 578, 1926.
† S. R. Pike, *ibid.*, **88**, 3, 1927.
‡ S. Chapman, *ibid.*, **89**, 458, 1929.

strongly concentrated round the normal or radial direction. If v_r, v_h are the radial and horizontal components of motion of a fast-moving atom, the horizontal acceleration, tending to increase the horizontal speed, is roughly proportional to $v_r v_h$, vanishing both for purely vertical and purely horizontal motion, and having its maximum when $v_r = v_h$, *i.e.* when the atom is moving at 45° to the vertical. In that case, however, the *radial* acceleration (f_r) exceeds four times the horizontal acceleration (f_h); in fact when $v_r = v_h$ Pike found that

$$f_h = 0{\cdot}0016\ v_r g, \qquad f_r = 0{\cdot}0068\ v_r g,$$

where g denotes solar gravity (0·27 km./sec.²). If

$$v_r = v_h = 50 \text{ km./sec.},$$
$$f_h = 0{\cdot}02 \text{ km./sec.}^2, \quad f_r = 0{\cdot}09 \text{ km./sec.}^2,$$

and in 10 minutes v_r will be increased to 104 km./sec., while v_h will be increased only to 63 km./sec., the inclination of the velocity to the vertical being thus reduced from 45° to 31°. Actually the reduction will be still greater, because as v_r/v_h increases f_h/f_r decreases; moreover, Pike's formula does not allow for the decrease of f_h relative to f_r with increasing height, which would certainly occur even if v_r/v_h remained the same. It is desirable to extend Pike's calculations to consider this effect, and to determine the probable frequency distribution, at a distance of several solar radii from the Sun, of atoms moving with different inclinations to the radius through the point of emission. In so doing it seems reasonable to assume that at the outset the emission of the fairly fast particles follows the cosine law, as in the case of light; if so, 82 per cent. of the atomic velocities would initially be inclined to the radial direction by less than 45°. In this case, though I have not made detailed calculations on the point, I think it probable that 90 per cent. of the atoms will finally have velocities inclined at less than 10° to the radius.

5. An important consequence of this is that at a great distance from the Sun—as, for example, at the Earth's orbit—the particles will form a nearly parallel beam, proceeding mainly from the solar area that (at the time of emission) was near the centre of the Sun's disc. In the case of light, the beam comes nearly equally from all parts of the Sun's disc, the foreshortening of the areas near the limb compensating for the reduced emission (according to the cosine law) at oblique angles; hence the Sun's disc appears nearly uniformly bright, there being only a small darkening towards the edge. But if we viewed the Sun with the aid of optical sensations due not to light but to the neutral corpuscles, the darkening towards the edge would appear far more pronounced, and indeed only the central area of the disc, subtending an angular radius of about 15° at the Sun's centre, or having an apparent diameter about one-fourth of that of the actual disc (*i.e.* about 7′ instead of 30′), would appear bright.

This near approach of the corpuscular rays to parallelism of velocity has the interesting consequence that, in an eclipse, the cone of total shielding from the corpuscles will, at the Earth, have a normal sectional diameter of

about 1600 miles, little less than the diameter of the Moon itself; while there will be but little penumbra or area of partial eclipse. If the apparent radius of the emitting area, for example, were exactly one-fourth that of the whole solar disc, the penumbra would have a width of only about 500 miles, instead of about 2000 miles, as for the eclipse of the Sun's light.

I have examined the effect of the gravitational deflection of the corpuscles in diminishing the diameter of the area of total corpuscular shadow, and find it to be negligible—the reduction of diameter being only about 1 mile.

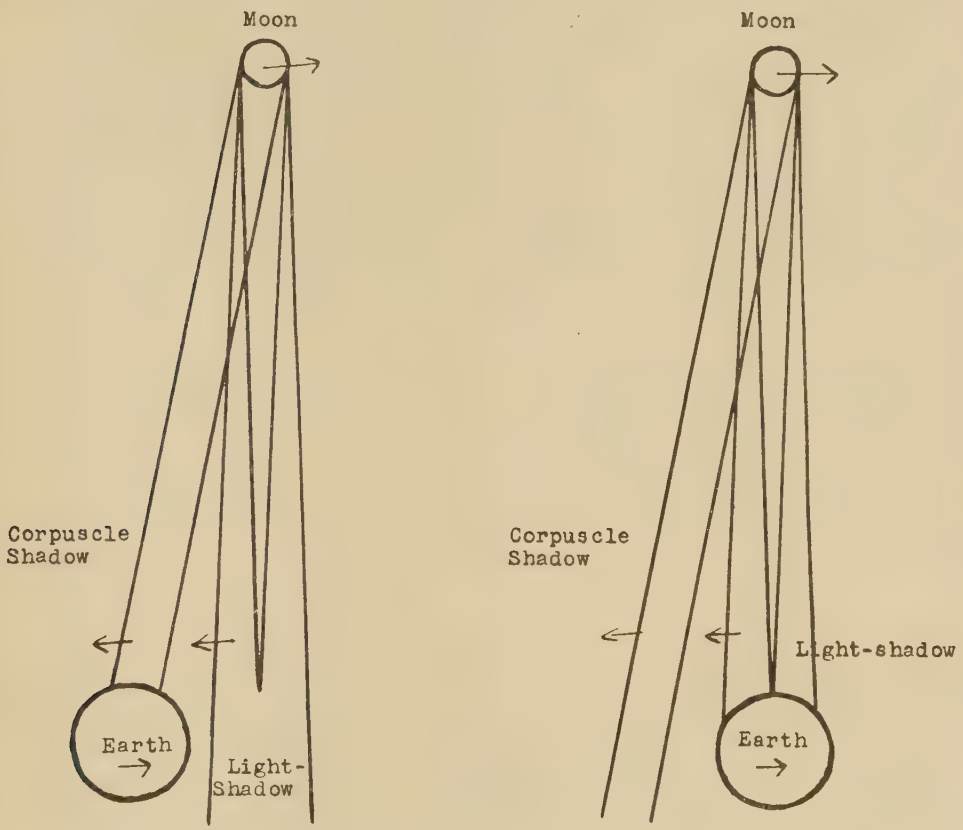
6. Thus the particles will cross the Earth's orbit in a direction nearly radial from the Sun; they are likely to have a small transverse velocity in the direction of the Earth's orbital motion, due to the solar rotation, but this velocity, at that distance from the Sun, will be only about 0·01 km./sec. and can be ignored. When the Moon is on the side of the Earth nearest the Sun it has a velocity of about 1075 miles/min. in the direction of the Earth's orbital motion. The particles therefore have an equal and opposite transverse velocity relative to the Moon, while their radial velocity is about 60,000 miles/min. (1600 kms./sec.) according to Milne. Hence it is easily seen that the shadow cylinder carved out by the Moon (treating the corpuscles as moving completely parallel to one another) will have a backward slope of approximately 1°. During the 4 minutes required for the particles to traverse the distance between the Moon and the Earth (240,000 miles), the transverse lag of the particles, or of the shadow cylinder, behind the Moon will be 4300 miles. Since the Moon, near the epoch of a solar eclipse, is moving with a velocity nearly parallel to that of the Earth but smaller by about 35 miles/min., it follows that the axis of the shadow cylinder of the particles will strike the Earth 4300/35 or 123 minutes before the axis of the optical shadow cone touches the Earth. Moreover, estimating the whole corpuscular shadow cast by the Moon as of 2600 miles diameter, it will take 2600/35 or 74 minutes for the corpuscular shadow cylinder to cross the centre of the Earth's disc.

The succession of shadow regions will therefore be as indicated in fig. 1, where, however, the angles and dimensions are incorrectly drawn to emphasize the points it is desired to bring out.

Taking a section of the shadow regions at the distance of the Earth's orbit, the distances are shown approximately in fig. 2 for a typical case. The distances are given in miles and also in minutes, at the rate of 35 miles per minute, representing the rate of travel of the shadow system relative to axes through the Earth's centre but not rotating with the Earth.

Owing to the Earth's rotation the actual time of passage of the shadow system past any terrestrial station will depend on the latitude, season and time of day at which the eclipse occurs at the station. At the equator near noon, for example, the peripheral speed of a point on the Earth, in the direction of the Earth's orbital motion, is 16 miles/min., so that the relative rate of motion of the shadow system is only 19 miles/min.; hence the corpuscular shadow system, about 2600 miles wide in all, would take about 135 minutes to traverse the station. But in high latitudes and near sunrise or sunset the component of the peripheral speed in the direction of the

Earth's orbital motion is much smaller, and the shadow system will therefore move with correspondingly greater rapidity. The sequence of events can be easily calculated for any given height above the Earth (a factor of which the importance in the case of the 1927 eclipse was pointed out by Dr. E. H. Rayner), for any actual eclipse, along the lines here indicated ; all that is



Corpuscular eclipse in progress, before light-eclipse begins.

Light-eclipse in progress, after corpuscular eclipse is over.

FIG. 1.

worth doing in this general discussion of the question is to indicate the main features in a typical case.

7. If the lower ionized layer owes its ionization to corpuscles of the kind here considered, the production of new ions will cease during the passage of the corpuscular shadow cylinder (ignoring the slight penumbra). Hence during this period the ion-content will steadily fall and minimum ion-content will occur at the *end* of the corpuscular eclipse. With the renewal of the corpuscular bombardment the ion-content will again begin to increase.

The total decrease in ion-content will depend on the duration of the eclipse (varying according to the season, time of day and the latitude), on the initial ion-content, and on the coefficient and mode of recombination. The probable order of magnitude can be roughly indicated in one or two typical cases. At the equator at noon the duration t of the eclipse is about 2 hours or 7000 seconds ; if the maximum electron-content n_0 is 10^6 per c.c., and the coefficient α of recombination (according to the law αn^2) is 10^{-9}, the electron-content n will be reduced to $n_0/(1+\alpha n_0 t)$ or $n_0/8$. At a station in a higher latitude or at a time near sunrise (or sunset), where t is only 1 hour and where n has the value 10^5 at the outset of the eclipse, the reduction will be only in the ratio $1/1{\cdot}35$; but if at the given time the value of n would normally be rising rapidly, the effect of the eclipse would be to produce this moderate decrease of n in place of the normal rapid increase ; in this case the effect of the eclipse might persist throughout the day and even be perceptible next morning. A corpuscular eclipse occurring near sunset would only accelerate the normal reduction of ion-content then experienced, and

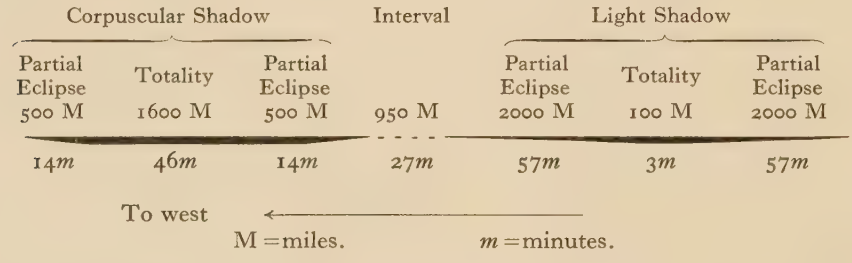


FIG. 2.

would therefore have a less important and less enduring influence on atmospheric ionization.

8. The influence of the eclipse of the Sun's ultra-violet light is rather different from that of the eclipse of the supposed solar particles, owing to the much greater penumbra in the former case. The reduction in the amount of ion-producing radiation q follows a more gradual course, of the form shown in fig. 4 (*a*), instead of (as in the corpuscular case) as in fig. 3 (*a*), where the eclipse is total for a much larger fraction of the whole eclipse. Hence the influence on the ion-content will be correspondingly smoother, as shown in fig. 4 (*b*), than in fig. 3 (*b*) for the corpuscular case. The curves are drawn for an eclipse occurring near noon, when q is nearly stationary ; similar curves for morning or afternoon eclipses can easily be drawn. I have made some rough calculations of the lag of the minimum of n after that of q, and found that it may be of the order of one-eighth of the duration of the eclipse (partial and total), though it depends on many factors that differ from one eclipse to another, and also on the recombination coefficient and the initial values of n and q. As stated in § 3, this time lag is about 15 minutes for an eclipse lasting 2 hours.

To sum up, the effect of the eclipse of the Sun's ultra-violet light will be to reduce the ionization of the atmospheric layer or layers which it ionizes ;

this reduction will be gradual, and greatest about 15 minutes after totality (or maximum phase in the case of a partial eclipse). The reduction will be experienced over a wide area, and the area and time of totality will not be distinguished from neighbouring regions and times by any marked discontinuity or gradient of ionization. The effects will depend greatly on the time of day at which the eclipse occurs, and also upon the latitude. Calculations of the expected effects must be made for each individual eclipse, as no one investigation can cover all cases even roughly. In such calculations the inclination of the Moon's orbit to the ecliptic must, of course, be taken into account. It must be remembered also that the interval between the corpuscular and the optical eclipse is not definitely calculable, since we do not know the speed of the corpuscles. If the radio observations show

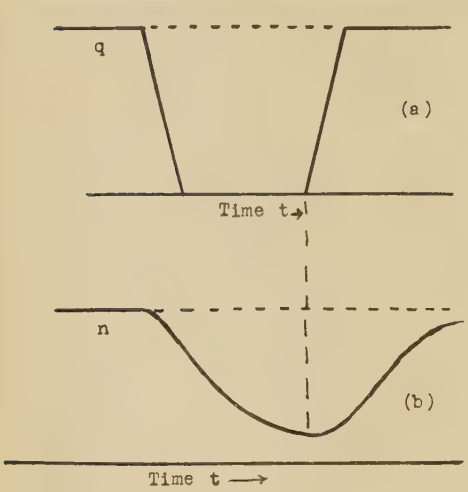


FIG. 3.—*Corpuscular eclipse.*

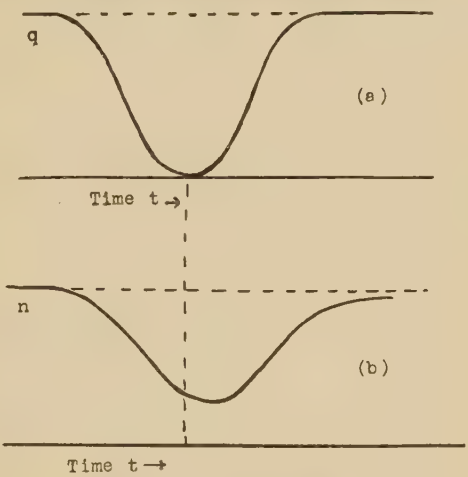


FIG. 4.—*Light eclipse.*

any signs of a corpuscular eclipse, besides throwing light on the origin of the ionization of the lower layer they will afford valuable information as to the speed of the particles.

9. Some remarks may be made about the eclipse of 1927 June 29, which passed over the lower ionized layer above England between 0520 and 0522 G.M.T. If this lower layer be ionized by corpuscles, the corpuscular eclipse must have commenced about 2 hours before optical totality—that is, at or about sunrise ; the sunrise effects might therefore not be experienced in the eclipsed region until the corpuscular eclipse was over. For some time before even the partial optical eclipse began, the layer should have been increasing in ion-content, though this would be still much below the normal value at that time of day. The greatest eclipse effects on the upper ionized layer must have been experienced further to the south, probably over the English Channel, owing to the much greater altitude of the layer ; the times of totality along this belt would be correspondingly slightly earlier.

The radio observations made on the day of the eclipse and the adjacent days for signals travelling in several directions relative to the eclipse track

showed some notable features nearly coinciding in time with the brief interval of totality. In the valuable account and discussion of the observations it was remarked * that "they constitute fairly strong evidence of a definite effect produced by the eclipse on the transmissions. At all these stations there is an outburst of variations during the half-hour of maximum observation, the centre of the outburst coinciding roughly with the time of totality." It is stated, however, that "unfortunately the possibility of this occurrence being somewhat of a coincidence cannot be entirely ruled out, as shown by the appearance of an outburst of the same nature (though certainly not quite so definite) on the day after the eclipse—about half an hour earlier."

I understand from Professor Appleton that this doubt extends only to part of the observations, and that he is inclined to interpret the 1927 eclipse results as showing that ultra-violet light is responsible for an important part of the ionization of the lower layer, if not for all of it.

It seems desirable in any case to take the opportunity afforded by the 1932 eclipse to settle the question definitely, and (though I cannot speak with expert knowledge on the experimental side) the best way would seem to be by measuring the changes in the maximum electron density, using the methods developed by Professor Appleton for this purpose.†

Note added during Proof-correction.—While writing the foregoing discussion, it had escaped my notice that the large time-interval between the corpuscular and optical eclipses would, owing to the diurnal rotation of the Earth, be accompanied by a considerable space-interval; the corpuscular eclipse track will lie to the east of the optical eclipse track, the interval in longitude corresponding approximately to the time-interval. I am indebted to Dr. Jackson and Professor Sampson for calling my attention to this important point.

Owing to the short time now available for preparation for observations of the 1932 August eclipse, I have asked Mr. J. C. P. Miller to add a map and brief description of the corpuscular eclipse track, calculated on the assumption that the corpuscular speed is 1600 km./sec. I am indebted to the Department of Scientific and Industrial Research for a grant for payment for this assistance.

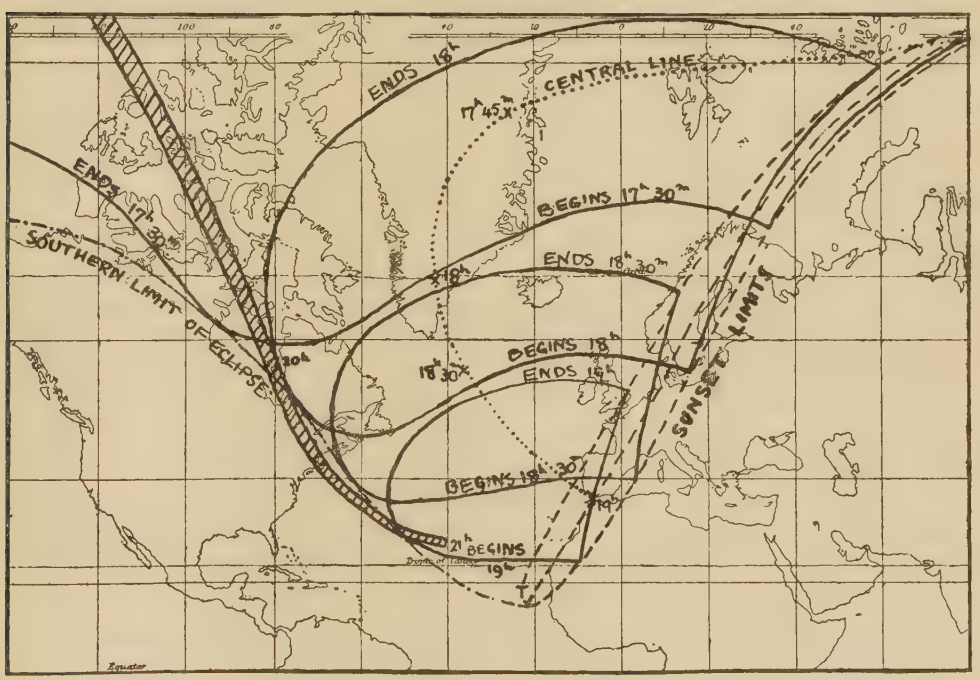
* "Wireless Observations during the Eclipse of the Sun, 1927 June 29," Dept. of Sci. and Ind. Research, *Radio Research Board Special Report*, No. 7, 1928, p. 22.

† E. V. Appleton, *Nature*, 1931 February 7.

MAP OF THE CORPUSCULAR ECLIPSE TRACK OF 1932 AUGUST.*

J. C. P. Miller, M.A.

The map is calculated on the assumption that the speed of the corpuscles is 1000 miles a second ; if the speed be less than this, the corpuscular eclipse track will be still farther to the east of the optical eclipse track.



The band of totality in the light eclipse is shaded on the map. Since the corpuscular speed is unknown, no great accuracy in the map of the corpuscular track has been attempted ; the errors should, however, be less than 200 miles in position and about 10 minutes in time. Times of beginning and ending of the eclipse have been rounded off to the nearest half-hour, but the error on this account is only 3 or 4 minutes.

The corpuscular eclipse track is in the position that the optical track would occupy, if made by a Sun 1°·05 behind the real Sun and a Moon about 2′ behind the real Moon. As the Moon, at the time of the eclipse, is approaching its descending node, the central line of the track is not only to the east, but also somewhat to the north, of the optical eclipse track.

* The " corpuscular eclipse " referred to is that dealt with in the previous paper by Professor S. Chapman.

The map shows the outline of the shadow at every half-hour from 17.30 to 19.00 G.M.T. The limits of the eclipse are also given, namely, the southern limit and the sunset limit. The shadows are calculated as if the Sun were a point (therefore giving no penumbra at all). This will not be strictly true, so that the umbra will be smaller than the shadows shown, while the eclipsed penumbral area will be greater.

According to the map, the central line of the corpuscular track runs across the Atlantic from Greenland to northern Africa. The shadow spreads from Canada to Europe, adjoining the belt of optical totality in eastern Canada, and passing across England shortly before sunset (it is to be noted that the fictitious Sun used in the calculations sets about 5 minutes before the real Sun). The eclipse passes over Iceland about half an hour before it reaches England, and the Sun is then at a considerably greater altitude.

ON THE USE OF COLOUR-FILTERS IN THE VISUAL PHOTOMETRY OF RED STARS.

M. G. J. Minnaert and J. van der Bilt.

1. *Introduction.*—A study of the light-fluctuations of U Cygni, based on observations of twenty members of the British Astronomical Association, encouraged a research the results of which have been set out in the present paper. As a general rule, estimates of brightness of a red variable star, made by various observers, show systematic differences, dependent on the star's brightness ; in the case of U Cygni, whose colour-index is about 4 magnitudes, these differences are rather startling.*

The problem presented by these discrepancies being of general interest, we have tried, by investigating a special case, to get a clearer insight into its causes. If this could be attained, advice might be given on how to reduce or annihilate the systematic differences between observers. The star chosen for this purpose is R Leonis, observed in 1931 by Messrs. F. de Roy (abbreviation DR) and J. van der Bilt (abbreviation Bl). DR used a 20-cm. reflector, Bl an 11·5-cm. refractor.

As a general rule, Bl sees a red variable star brighter than DR. In the case of U Cygni the systematic difference increases from $0^m{\cdot}2$ at magnitude 11 to $1^m{\cdot}0$ at magnitude 6·5 ; in the case of R Leonis it increases from $0^m{\cdot}1$ to $0^m{\cdot}5$ between the magnitudes 10 and 6. It is evident that the numerical differences between these two cases must result from a difference of colour, U Cyg being much redder than R Leo.

2. *Laboratory Experiments.*—In order to know how far the instruments employed must be held responsible for the facts, we have submitted the observers to a laboratory experiment. For this purpose, Mr. de Roy has kindly consented to come from Antwerp to Utrecht. Artificial stars were obtained by illuminating (*via* a white screen and a reflecting prism) two

* The work on U Cygni by J. van der Bilt will be shortly published.

minute pin-holes made in a sheet of stanniol. These "stars" were placed in the focus of a collimator and could be observed in a small telescope.

First Experiment.—Both screens were illuminated by an automobile lamp, and the observers were requested to mark the moment of equal brightness, when one of the lamps was put nearer to, or farther away from, the corresponding screen. Their results did not differ by more than 3 per cent. ($=0^m\cdot03$).

Second Experiment.—A red-glass filter was placed in front of one of the

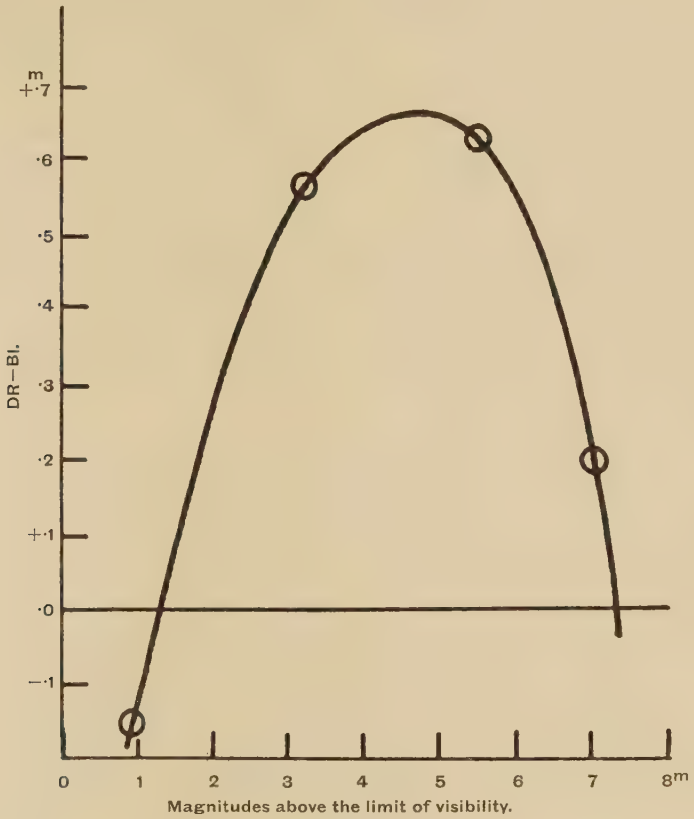


FIG. 1.

lamps and the observers were again requested to mark the moment of equal brightness. This experiment was made at four values of the brightness, which the observers estimated to lie $0^m\cdot85$, $3^m\cdot25$, $5^m\cdot5$ and about 7 magnitudes above the limit of their instrument. The results, which are shown in fig. 1, are in accordance with those derived from the light-curves of U Cyg and R Leo ; the difference is small when the stars are very faint, and increases with increasing brightness, Bl seeing the red star brighter than DR. But a new fact is shown, viz. that the estimates of the two observers approach one another again beyond a certain brightness.

These few results had necessarily to be based on a very limited number of observations, and they therefore possess only a qualitative value.

3. *A Working Hypothesis.**—We shall suppose that the systematic differences between the two observers result from their making different uses of the cones and the rods in the retinas of their eyes. The cones are adapted to a bright illumination and have their maximum sensitivity in the yellow ; the rods are adapted to a feeble illumination and are most sensitive to green-blue.

Faint images must, of necessity, be observed by both observers with their rods, since the light is too feeble to stimulate the cones ; therefore the observers will agree when comparing a white and a red star, both of low luminosity. With increasing brightness, Bl will be using his cones long before DR uses his ; and the result must be that Bl will estimate a red star, as compared with a white star, to be brighter than will DR. After a brightness has been reached at which DR brings his cones into action, the difference will gradually fade away and finally the observers will again agree. This hypothesis easily explains the facts, observed with both real and artificial stars. In short terms we will say that the observers have different "retina-activities." The process suggested above may result from the habit of the observer of either fixing the image or, in an unconscious way, looking aside of it. In the first of these cases the light is received on the yellow spot of the retina and the cones become active. In the second case the light is received by the periphery of the retina and only the rods become active. Between these two extremes several intermediate cases of retina-activity may be imagined.

Two other observers have been submitted to the same laboratory experiments. The differences between their estimates were of the same kind as those between DR and Bl ; afterwards, from a description of their way of observing, it became evident that the directions in which they had looked had not been the same for both. Few observers seem to have a good judgment about what they are exactly doing, and many observers, on being told that they are likely to look *beside* the object, refuse to believe it.† The inclination to look aside will be greater when objects of different colours must be compared, because they become "achromatized," and their brightness therefore more easily comparable, when they are not fixedly regarded.

The Purkinje effect, according to modern opinion, disappears when starry images are steadily fixed.‡ Whether this effect for such images becomes reversed,§ and whether the additive character of intensities will strictly hold for them,‖ are two problems which we have not considered. Little is known about them, and the influence on our results must, anyway, be small.

We close this paragraph by stating that our working hypothesis was not mentioned to Mr. de Roy till after the laboratory experiments, so he could not have been prejudiced in his mode of observing.

* The interpretation of the Purkinje effect and related questions, given in this paragraph, is in broad lines concordant with modern views on visual photometry. (See, *e.g.*, W. Hassenstein, *Handbuch der Astrophysik*, II_2, Kap. 6.)

† *Cf.* W. Nagel in Helmholtz, *Hdb. d. Physiolog. Optik*, 3rd ed., **2**, 281.

‡ W. Nagel, *ibid.*, p. 305.

§ Ch. Galissot, *La photométrie du point lumineux*, Thèse, Lyon, 1921 ; A. Danjon, *Ann. Obs. Strasbourg*, **2**, 12, 1928.

‖ J. Escher Desrivières, *Rev. d'opt.*, **10**, 389, 1931.

4. *Observations with Colour-filters.*—If our theory is right, the systematic differences between the observers are bound to disappear when use is made of monochromatic filters. Since the available glass filters transmit a rather large portion of the spectrum, these differences can only be reduced to a certain extent. This we decided to verify by the observations of R Leo, mentioned before. The star, at the beginning of 1931, was on its way to maximum brightness, and half-way on a "stillstand" of remarkably long

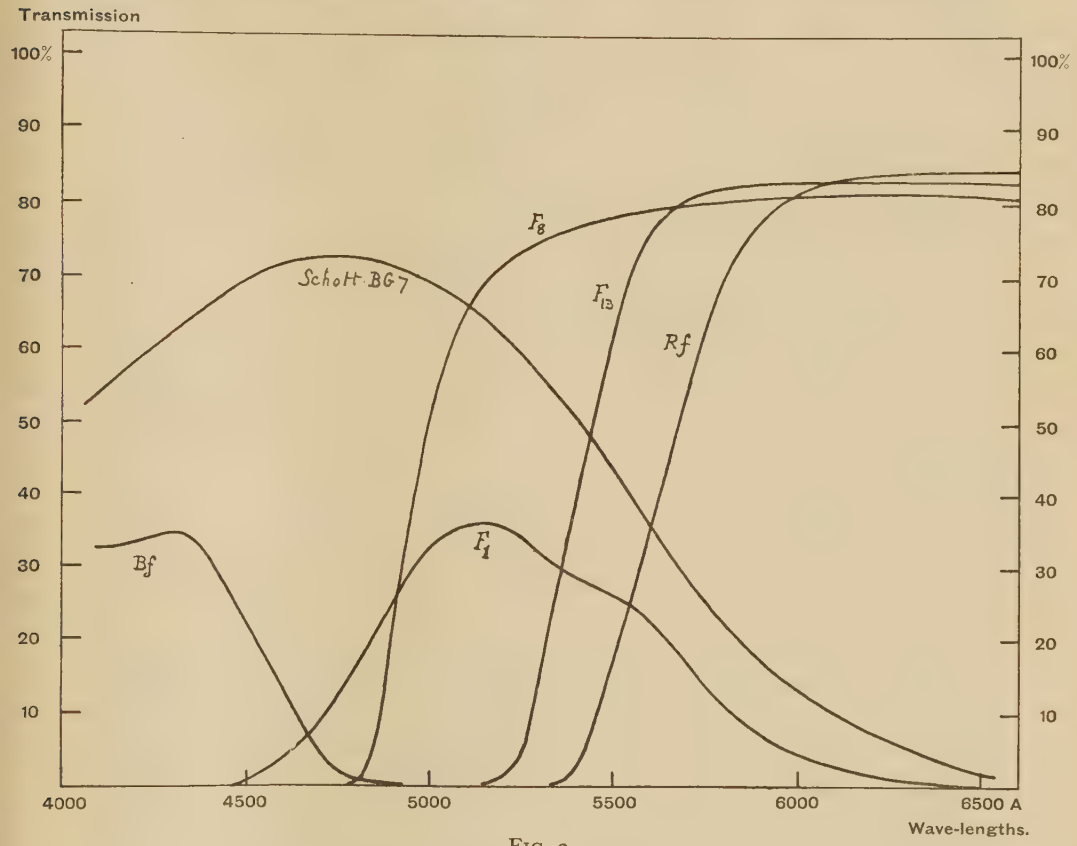


FIG. 2.

duration.* The maximum brightness could just be observed before the star disappeared in the evening sky. The interval of time, though short, was long enough to enable us to judge of the value of filter-observations and of our method of handling them.

Five colour-filters were used ; the red and the blue filters (Rf and Bf) were gelatine films from Steeg und Reuter, the orange, yellow and green filters (F_{13}, F_8 and F_1) were glass filters of unknown origin, in the possession of the physical laboratory at Utrecht. The transmission curves, obtained with a monochromator, a thermopile and a galvanometer with thermo-relays are shown in fig. 2. Reflection has been taken into account. Both

* See also Report of the Director of the Variable Star Section of the B.A.A. in *J.B.A.A.*, 42, 21, 1931.

observers had trouble with the blue filter, through which the variable could only be seen near maximum ; the red filter, too, was a difficult one.

5. *The Principle of Heterochrome Photometry.*—The *physiological intensity*, which we attribute to a star seen through a colour-filter, may be given by the expressions :

$$\mathcal{J}(c, f) = s\int_0^{+\infty} i_\lambda c_\lambda f_\lambda d\lambda \text{ for the cones,}$$

and

$$\mathcal{J}(r, f) = s\int_0^{+\infty} i_\lambda r_\lambda f_\lambda d\lambda \text{ for the rods.}$$

The corresponding "brightness" will be :

$$B(c, f) = 2{\cdot}5 \log \mathcal{J}(c, f) \quad \text{and} \quad B(r, f) = 2{\cdot}5 \log \mathcal{J}(r, f).$$

The symbols have the following meanings :

$i_\lambda \, . \, d\lambda$ = the spectral intensity at λ ;
c_λ, r_λ = the sensitivity of cones and rods respectively ;
f_λ = the transmission of the filter ;
s = a constant, depending on the star's magnitude but independent of λ and f.

For observations in white light (WL-observations) the corresponding expressions are :

$$\mathcal{J}(c, w) = s\int_0^{+\infty} i_\lambda c_\lambda d\lambda \; ; \qquad B(c, w) = 2{\cdot}5 \log \mathcal{J}(c, w) \; ;$$

$$\mathcal{J}(r, w) = s\int_0^{+\infty} i_\lambda r_\lambda d\lambda \; ; \qquad B(r, w) = 2{\cdot}5 \log \mathcal{J}(r, w).$$

Since the observations of R Leo have not been made near the horizon, selective extinction has been neglected ; nor have we considered the selective reflection of DR's speculum and the absorption in Bl's objective glass. They must all be small, compared with the rather high selectivity both of the filter-transmission and of the sensitivity of the eye.

Our first step, now, will be to derive for the comparison stars of R Leo, their magnitudes $B(c, f)$ and $B(r, f)$ as seen through each of the filters. With the exception of Hagen's numbers 16 and 21, the spectra of these stars could be taken from the Draper Catalogue. For 16 and 21 we took the photographic magnitudes from the Carte du Ciel and reduced them by a graphical process to the Harvard photographic system ; the visual magnitudes in the same system being known, we got a fairly reliable result for the colour-indices and, hence, for the spectra.

The quantities i_λ could not be taken from the black-body radiation curves, since, chiefly through the presence of Fraunhofer lines, the true distribution of energy differs too much from these curves. So we had recourse to the tables of Rosenberg and Wilsing, as revised by A. Brill.* Rosenberg's tables were used from 4000 A. to 4880 A.; those of Wilsing from 4510 A. to 7000 A. In the overlapping region means were taken.

* *A.N.*, **219**, 21, 1923, Tables IX and XIV.

The i-curves, so obtained, sometimes show a lack of smoothness, which must have caused some apparent irregularities in the tables which we shall give later on.

The quantities c_λ and r_λ were taken from the tables adopted by the International Commission on Illumination * and from A. Kohlrausch's measurements.† *Absolute* values of these quantities cannot be given; for cones, as well as for rods, the values given are relative to an arbitrary value 100 at maximum sensitivity. However, though the impression made by *one* particular star upon the retina of a c-observer cannot be compared with that made upon the retina of an r-observer, the former will register a *difference in brightness between two stars*, which is comparable with that registered by the latter. In order to be free from *a priori* suppositions we have, for each observer, made all further reductions twice, viz. (1) supposing him to use exclusively his cones (for brevity we shall introduce the expression "c-observer"); and (2) considering him to be an "r-observer."

Our method of reduction does not require a knowledge of the constants s. They might easily have been determined from the condition that, for white light, the intensity-integral of each comparison star multiplied by s and reduced to brightness, should yield the right photometric magnitude in the Harvard system.

6. *An Explanation of the Difference between the WL-curves of the two Observers.*—Tables I*a* and I*b* contain the fundamental quantities c_λ, r_λ and $c_\lambda i_\lambda$, $r_\lambda i_\lambda$ for every spectral type present among the comparison stars and for a black body at three different temperatures. Being independent of the observer, these values may be useful to others in future work along the same lines. The products have been read from a slide-rule and are liable to errors of a few parts in a thousand. Summation has been substituted for integration; this gives a very satisfactory approximation.

The sums represent the integrals of the physiological intensity, $\mathcal{J}/s$. The corresponding brightnesses, $2{\cdot}5 \log \mathcal{J}/s$, are given on the lowest line; since the factor s is involved, they contain an arbitrary constant, the same for both cones and rods, but differing from star to star.

The differences $B(c, w) - B(r, w)$ represent, for each spectral type, to what extent an r-observer will see a star of this type fainter than a c-observer. [We may, once for all, emphasize that, strictly speaking, our way of expressing this is illegitimate, but that it does not lead to erroneous conclusions because in practice we are handling only *differences* of brightness. Therefore, unknown constants, which should reduce c- and r-brightnesses to "absolute" scales, necessarily disappear.] Apart from an arbitrary additive constant, these values are:

for	Ao	Fo	F2	F8	Go	G5	Ko	K5	Mo	3200°	2700°	2000°
	$0^m{\cdot}15$	·19	·21	·25	·28	·33	·38	·47	·54	·60	·73	$1^m{\cdot}03$

Since nearly all the comparison stars of a red variable will have spectra

* K. S. Gibson, E. P. T. Tyndall, *Sc. Papers Bur. of Stand.*, No. 475, **19**, 131, 1923; J. T. W. Walsh, *Journ. Opt. Soc. Amer.*, **11**, 111, 1925.

† *Pflügers Archiv*, **200**, 215, 1923.

between Ao and Mo, we may say that an *r*-observer will in the mean see them $0^m.3$ fainter than a *c*-observer. For the variable itself this difference is much larger. If we assume the temperature of R Leo to be 2350° (see later), an *r*-observer will see it $0^m.9$ fainter than a *c*-observer. Relative to

Table I*a*

λ	c_λ	$c_\lambda i_\lambda$											
		Ao	Fo	F2	F8	Go	G5	Ko	K5	Mo	3200°	2700°	2000°
4100	0·08	0·1	0·1	0·1	0·1	0·1	0·0	0·0	0·0	0·0	0·02	0·01	0·00
4200	0·50	0·8	0·6	0·5	0·4	0·4	0·3	0·2	0·1	0·1	0·15	0·09	0·03
4300	1·2	1·8	1·4	1·3	1·1	1·0	0·8	0·6	0·4	0·4	0·40	0·25	0·09
4400	2·3	3·3	2·6	2·5	2·2	2·0	1·7	1·4	1·1	0·9	0·86	0·58	0·23
4500	3·8	5·2	4·3	4·2	3·7	3·5	3·1	2·6	2·2	1·9	1·60	1·11	0·49
4600	6·0	7·8	6·8	6·6	6·0	5·7	5·2	4·6	3·9	3·6	2·81	2·03	0·99
4700	9·1	11·2	9·0	9·9	9·2	9·0	8·1	7·3	6·4	6·2	4·70	3·53	1·88
4800	13·9	16·4	15·2	14·9	14·0	14·0	12·8	11·8	10·5	10·4	7·89	6·17	3·56
4900	20·8	23·5	22·3	21·8	20·8	20·6	19·1	18·2	16·5	16·1	12·90	10·40	6·50
5000	32·3	34·4	33·4	33·4	32·3	31·3	29·5	28·4	26·3	25·2	21·80	18·20	12·20
5100	50·3	53·0	51·7	51·3	50·9	48·5	46·0	44·5	42·3	40·0	36·60	30·30	22·80
5200	71·0	74·4	72·2	72·2	71·7	68·5	66·5	64·3	62·9	59·1	55·50	49·55	38·50
5300	86·2	86·2	87·5	86·2	86·2	84·6	84·7	82·1	82·3	82·3	72·20	66·20	54·90
5400	95·4	90·2	95·4	93·0	93·7	93·7	94·5	94·7	95·4	97·1	85·00	80·70	71·10
5500	99·5	89·1	95·0	93·3	95·0	96·0	98·6	99·5	102·2	106·1	94·00	91·50	86·00
5600	99·5	84·6	89·9	90·4	93·3	94·0	97·7	100·0	104·0	109·0	99·50	99·50	99·50
5700	95·2	77·4	83·0	83·7	86·9	88·3	92·8	96·2	102·3	107·3	100·40	102·70	109·10
5800	87·0	69·1	72·9	74·5	78·0	79·2	84·0	78·9	94·6	100·6	96·30	101·30	113·70
5900	75·7	58·6	61·3	62·9	66·0	67·6	72·1	76·4	82·8	89·1	87·20	94·30	111·50
6000	63·1	47·0	49·7	50·7	54·0	55·5	59·1	63·7	70·0	76·0	76·30	84·10	104·90
6100	50·3	36·0	38·6	39·2	42·0	43·4	46·6	50·8	56·0	62·1	63·10	71·50	93·80
6200	38·1	26·1	28·3	28·6	31·0	32·0	34·8	38·5	42·5	47·5	49·80	57·40	79·00
6300	26·5	17·6	19·0	19·3	21·3	21·8	23·6	26·5	29·5	33·3	35·80	42·15	60·90
6400	17·5	11·2	12·2	12·5	13·6	14·0	15·5	17·4	19·6	22·5	24·50	28·30	44·40
6500	10·7	6·6	7·2	7·1	8·2	8·4	9·3	10·5	11·9	14·2	15·30	18·80	29·80
6600	6·1	3·6	3·9	4·1	4·6	4·7	5·3	6·0	6·9	8·2	9·00	11·30	18·70
6700	3·2	1·8	2·0	2·1	2·3	2·4	2·7	3·1	3·6	4·4	4·85	6·19	10·70
6800	1·7	0·9	1·0	1·1	1·2	1·3	1·4	1·7	2·0	2·4	2·65	3·44	6·17
6900	0·82	0·4	0·5	0·5	0·6	0·6	0·7	0·8	1·0	1·2	1·30	1·72	3·21
7000	0·41	0·2	0·2	0·2	0·3	0·3	0·3	0·4	0·5	0·6	0·67	0·90	1·74
$\mathcal{J}/s =$		938·5	967·2	968·1	990·6	992·4	1016·8	1031·1	1079·7	1127·8	1063·10	1084·22	1186·40
		$7^m.43$	$7^m.46$	$7^m.46$	$7^m.49$	$7^m.49$	$7^m.52$	$7^m.53$	$7^m.58$	$7^m.63$	$7^m.57$	$7^m.59$	$7^m.69$

the comparison stars, the difference becomes $0^m.9 - 0^m.3 = 0^m.6$. Now, from a long series of observations of R Leo by DR and Bl, published in the *Memoirs of the B.A.A.*, we could represent the systematic difference, DR – Bl, by a curve, which has its maximum at $+0^m.56$. This figure can now be entirely explained on the supposition that DR is chiefly an *r*-observer and Bl chiefly a *c*-observer. The same explanation will hold for U Cyg, but since for this star the observations showed a maximum systematic

difference, DR – Bl = 1^{m}·0, its temperature must be much lower than 2000° (probably about 1600°) ; the same result has been found from a preliminary discussion of filter-observations of this star.

TABLE I*b*

λ	r_λ	$r_\lambda i_\lambda$											
		Ao	Fo	F2	F8	Go	G5	Ko	K5	Mo	3200°	2700°	2000°
4300	4·0	6·0	4·5	4·3	3·5	3·3	2·7	2·0	1·4	1·3	1·33	0·85	0·31
4400	9·0	12·8	10·2	9·8	8·4	8·0	6·7	5·5	4·2	3·4	3·39	2·30	0·91
4500	15·0	20·3	19·1	16·6	14·6	13·8	12·2	10·2	8·5	7·4	6·32	4·39	1·95
4600	22·4	29·0	25·3	24·6	22·3	21·2	19·4	17·1	14·5	14·5	10·50	7·56	3·70
4700	33·7	41·6	33·5	36·8	34·2	33·5	30·1	27·1	23·8	22·9	17·40	13·08	6·94
4800	45·0	53·1	49·2	48·2	45·3	45·5	41·4	38·2	34·5	33·5	25·50	19·97	11·52
4900	56·0	63·1	60·0	58·7	56·1	55·6	51·5	49·1	44·5	43·3	34·80	28·00	17·50
5000	76·0	81·2	78·2	78·2	75·8	74·0	69·1	66·7	62·1	59·2	51·10	42·90	28·70
5100	93·6	98·8	101·2	100·5	99·4	90·2	90·1	87·1	78·6	74·2	68·00	56·30	42·40
5200	100·0	105·0	102·0	102·0	101·0	96·4	93·7	90·8	88·7	83·2	78·20	69·90	54·10
5300	95·1	95·1	96·9	95·3	95·3	93·3	93·6	91·0	90·8	90·9	79·70	73·10	60·40
5400	81·3	76·9	81·4	79·2	79·8	79·9	80·7	80·7	81·3	83·0	72·40	68·80	60·40
5500	61·1	54·9	58·4	57·3	58·4	59·0	60·7	61·1	63·0	65·3	57·80	56·20	52·90
5600	41·1	35·0	37·1	37·3	38·6	38·9	40·3	41·3	43·0	45·1	41·10	41·10	41·10
5700	26·7	21·7	23·2	23·4	24·2	24·8	25·9	27·0	28·7	30·1	28·20	28·80	30·60
5800	16·3	12·9	13·6	13·9	14·6	14·9	15·7	14·8	17·7	18·9	18·00	19·00	21·30
5900	9·1	7·1	7·4	7·5	7·9	8·1	8·6	9·2	10·0	10·7	10·50	11·35	13·40
6000	4·7	3·5	3·7	3·8	4·0	4·1	4·4	4·7	5·2	5·7	5·69	6·27	7·81
6100	1·7	1·2	1·3	1·3	1·4	1·5	1·6	1·7	1·8	2·1	2·14	2·27	3·16
6200	0·3	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·3	0·4	0·39	0·45	0·62
$\mathcal{J}/s$ =		819·4	806·4	798·9	785·0	766·2	748·7	725·6	702·6	695·1	612·46	552·59	459·72
		7^{m}·28	7^{m}·27	7^{m}·26	7^{m}·24	7^{m}·21	7^{m}·19	7^{m}·15	7^{m}·12	7^{m}·11	6^{m}·97	6^{m}·86	6^{m}·66

7. *The Reduction of the Filter-observations. The Temperature of R Leonis.* —Observation through a filter reduces a star's brightness from $B(c, w)$ to $B(c, f)$, or from $B(r, w)$ to $B(r, f)$. The reduction amounts to :

$$2{\cdot}5 \log \frac{\int_0^{+\infty} i_\lambda c_\lambda d\lambda}{\int_0^{+\infty} i_\lambda c_\lambda . f_\lambda d\lambda} \quad \text{for a } c\text{-observer,}$$

and a similar expression for an *r*-observer. These quantities are given in Table II. When we add them to the Harvard photometric magnitudes * we get the brightness of the comparison stars as seen through each of the filters.

* These magnitudes should have been revised after a comparison with the observer's scale in steps ; the effect of this process was so small that we decided to stick to the direct Harvard magnitudes.

Table II

Sp.	$B(c,w)-B(c,f)$					$B(r,w)-B(r,f)$				
	Bf	F_1	F_8	F_{13}	Rf	Bf	F_1	F_8	F_{13}	Rf
	m	m	m	m	m	m	m	m	m	m
Ao	5·77	1·74	0·37	0·77	1·20	4·22	1·49	0·72	1·89	2·95
Fo	5·93	1·75	0·35	0·73	1·16	4·26	1·47	0·63	1·81	2·87
F2	6·10	1·76	0·36	0·73	1·16	4·30	1·47	0·65	1·79	2·80
F8	6·20	1·78	0·35	0·70	1·12	4·56	1·46	0·64	1·76	2·79
G5	6·36	1·83	0·34	0·66	1·06	4·72	1·47	0·61	1·66	2·63
Ko	6·67	1·84	0·33	0·64	1·02	4·87	1·46	0·60	1·63	2·62
K5	6·95	1·87	0·32	0·60	0·96	5·03	1·45	0·56	1·54	2·50
Mo	7·09	1·91	0·31	0·57	0·93	5·11	1·44	0·54	1·47	2·42
3200°	7·18	1·95	0·31	0·54	0·87	5·18	1·47	0·63	1·45	2·36
2700°	7·59	2·03	0·29	0·49	0·79	5·45	1·46	0·49	1·34	2·22
2000°	8·53	2·21	0·27	0·41	0·65	6·09	1·48	0·42	1·14	1·94

For each observer the estimated brightness of R Leo, as seen through the filters, could now be obtained by interpolation. So we got, for both DR and Bl, 10 "coloured light"-curves, 5 on the supposition that they are *c*-observers and 5 on the supposition that they are *r*-observers. As an example we give in the upper part of fig. 3 the combined result yielded by the green filter F_1 (both observers considered as *c*-observers); and in the lower part the two WL-curves. It is at once apparent that the agreement between the two observers has been materially improved by the use of the filter.*

Table III

Star	$B(c,w)-B(c,f)$					$B(r,w)-B(r,f)$				
	Bf	F_1	F_8	F_{13}	Rf	Bf	F_1	F_8	F_{13}	Rf
	m	m	m	m	m	m	m	m	m	m
R Leo (DR)	7·16	1·95	0·18	0·34	0·73	5·54	1·63	0·44	1·38	2·31
R Leo (Bl)	7·82	2·27	0·37	0·54	0·63	6·08	1·95	0·63	1·55	2·26
3200°	7·18	1·95	0·31	0·54	0·87	5·18	1·47	0·63	1·45	2·36
2700°	7·59	2·03	0·29	0·49	0·79	5·45	1·46	0·49	1·34	2·22
2000°	8·53	2·21	0·27	0·41	0·65	6·09	1·48	0·42	1·14	1·94

* Near the maximum, where Bl has only two observations, the differences between the WL do not run parallel with the mean results derived from a long series of observations.

We have divided the filter-observations into three groups, according to the phase of the light-variation. The mean difference between the brightness in WL and that in coloured light $B(w) - B(f)$, was derived for each separate group and for each of the observers. Each group gave practically the same mean results. The differences were of the same order as the errors of observation ; so nothing could be learned about possible spectral changes along the observed part of the light-curve. We have, finally,

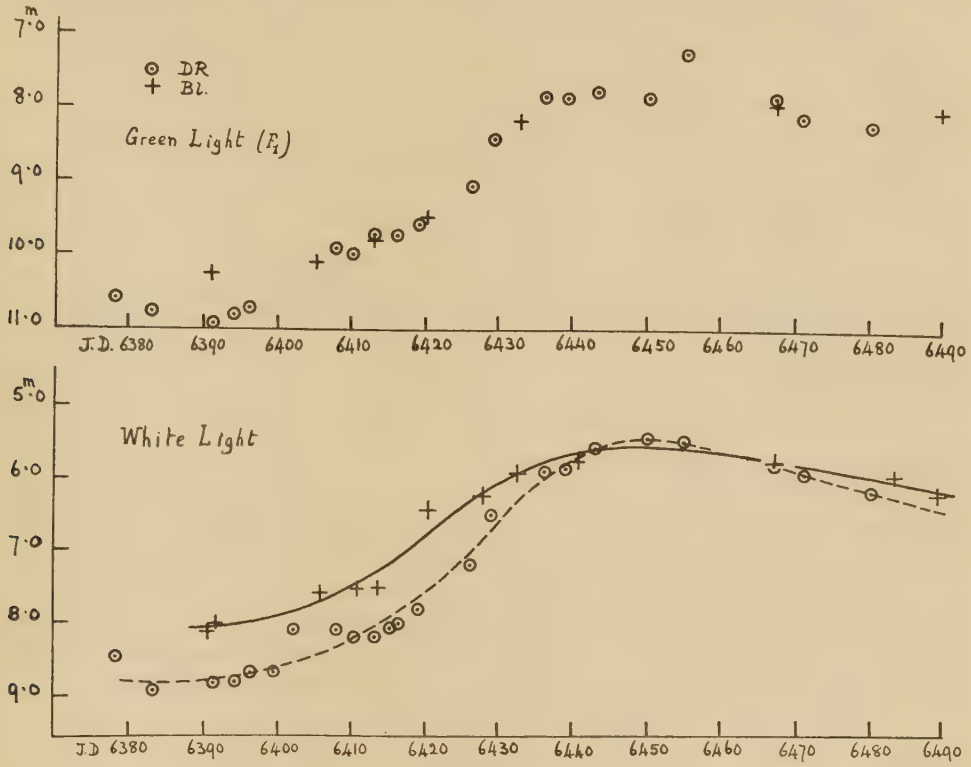


FIG. 3.

taken all groups together ; the results are given in the first two lines of Table III. They contain, for both observers, a double set of differences, $B(w) - B(f)$.

The next step was *to investigate at what temperature black-body radiation will give these same differences.**

The last three lines of Table III give the values of $B(w) - B(f)$ for the temperatures 3200°, 2700° and 2000° respectively. From these numbers no immediate conclusion as to the temperature can be drawn, since the

* The energy-curve of R Leo will probably differ appreciably from that of a black body ; however, in absence of spectrophotometric measurements for M8 stars, we have no alternative. It renders the value of " temperature " rather questionable.

question, cones *versus* rods, is involved in the problem. As to this, we have considered five possibilities :

a. the observer is a 100 per cent. *c*-observer (retina-activity = 100).
b. ,, ,, is a 75 per cent. *c*- ,, and 25 per cent. *r*-observer.
c. ,, ,, is a 50 per cent. *c*- ,, ,, 50 per cent. *r*- ,,
d. ,, ,, is a 25 per cent. *c*- ,, ,, 75 per cent. *r*- ,,
e. ,, ,, is a 100 per cent. *r*- ,,

We do not think that there is any objection to making such a hypothesis. The spot on the retina where the light is received may be a mosaic of cones and rods ; or the observer may be supposed, while making his estimate, to receive the impression a certain number of times upon, and a different number of times beside, the yellow spot. The differences $B(w) - B(f)$ for the intermediate cases have been simply obtained by interpolation between the two extreme cases.

Our problem, then, will be this : *which combination of temperature and retina-activity gives results agreeing best with the observed differences ?*

The diagrams in fig. 4 and the "constellations" formed by connecting the points may be useful in making our choice. The five filters are represented by points (equally spaced) on the line of abscissæ. Ordinates are the quantities

$$[B(w) - B(f)] \text{ observed} \quad \textit{minus} \quad [B(w) - B(f)] \text{ computed}$$

for the three temperatures considered.

An ideal solution would be a temperature T and a "retina-activity" α per cent. which would make all the ordinates zero. Since a change of temperature must cause a rotation of the "constellation," and apparently a change in α causes an upward or downward translation, the practical solution will be a temperature T and a value of α which renders the median line of the constellation parallel to and at the shortest distance from the line of abscissæ. One look at the diagrams shows that these conditions are best fulfilled

for DR at $T = 2700°$ and $\alpha = 35$,

and for Bl at $T = 2350°$ and $\alpha = 100$.

The result for T is in fair agreement with the value 2260°, obtained by E. Pettit and S. B. Nicholson from heat-index and water-cell absorption.* Their result, too, must be uncertain owing to the fact that the star does not emit a black-body radiation. Our result shows, at any rate, that a direct estimate of the temperature of such a star may result from a few simple observations of its brightness through colour-filters.

Our result for α means an independent confirmation of the results yielded by the WL-curves and the laboratory experiment (see p. 422). From a longer series of observations it would probably be possible to show a dependency of α on the star's brightness.

8. *The Reduction of Filter-observations to "Normal Magnitudes."*—A further problem is how to apply the results obtained with colour-filters to previous observations of the same star ; and, secondly, how to combine the estimates

* *Astroph. Journ.*, 68, 291, 1928.

of observers using different filters. The problem will be solved if we find a way to reduce all such observations to white light and a "normal" eye. As such we propose to adopt the eye of a *c*-observer, with a sensitivity c_λ, which has the same dependency on λ as that which occurs in the measurements of the International Committee on Illumination and which we have adopted in the present investigation (see p. 427).

For an *exact* solution of this reduction problem we must know the star's T and the observer's α. We have found approximate mean data for the special case considered, but generally T will be known only roughly, and

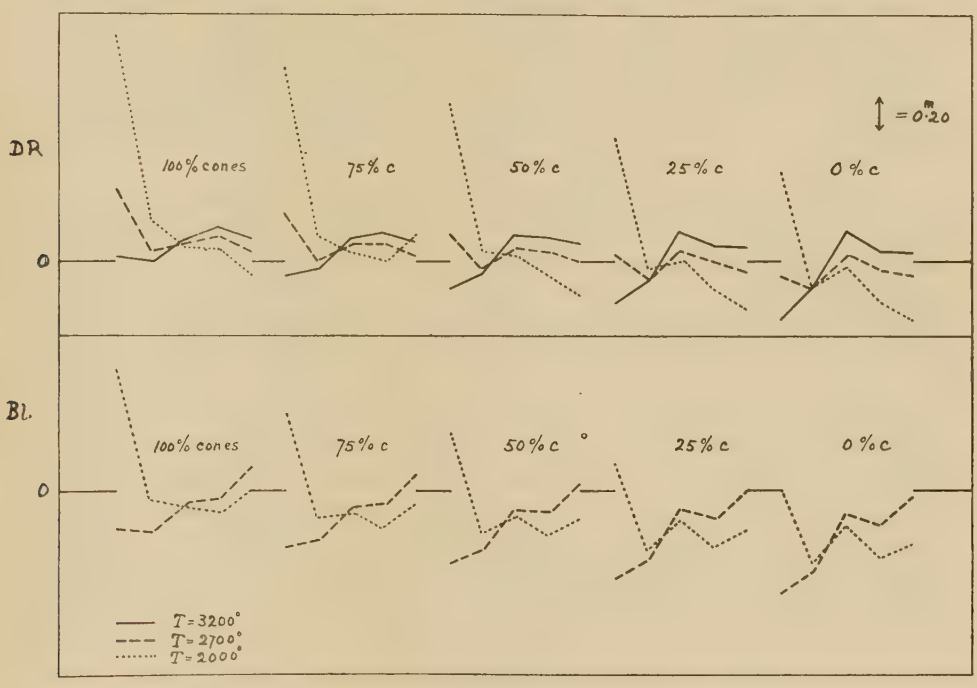


Fig. 4.

α not at all. Furthermore, in order to save time, future observations will probably be made with a single filter, which will make it impossible to get α and T from such diagrams as those shown in fig. 4. We must, however, keep in mind that observations with any filter, *sufficiently monochromatic*, must lead to the same "normal brightness," whatever the individual properties of the observer's eye may be; and, moreover, that an exact knowledge of T will not be needed. These considerations will allow us to give a simpler method of reduction for practical use.

A. *The Exact Reduction applied to R Leonis.*—The observations with the blue filter were few in number and so difficult that the mean observational errors largely exceeded those for the other filters. Therefore they were excluded from the present reduction.

The observer Bl was found to be a 100 per cent. *c*-observer. Therefore the mean value of $B(c, w) - B(c, f)$ from Table II could be directly applied

to his filter-observations. The "normal-eye"-curve, which resulted, differed but very little (about $0^m.06$) from Bl's WL-curve. This would be expected, since the reverse way had been trodden before ; the condition, small differences $B(w) - B(f)$, led to $\alpha = 100$.

The observer DR has $\alpha = 35$. All reductions were made for $\alpha = 100$, then for $\alpha = 0$ (which involved the changes in the brightness of the comparison stars previously mentioned, and finally, by interpolation, for $\alpha = 35$. For him, the resulting "normal-eye"-curve differed to a large extent from his own WL-curve. This, too, is natural, since his eye in no way answers to what we have defined as "normal."

The chief result of these reductions is that *the "normal-eye"-curves of both observers agree with each other in a very satisfactory way.* The mean difference is only $0^m.12$, and this agreement is *not* brought about by suppositions *ad hoc*. It could hardly be expected to be closer, considering (1) that the filters were not extremely monochromatic ; (2) that the value of α is an average one ; and (3) that the observations had their errors.

B. *The Practical Reduction applied to R Leonis.*—Assuming that red variables will, in future, be observed with *one* filter only, we have investigated whether a reliable "normal-eye"-curve can be derived in this restricted case. We have chosen the green filter F_1 to serve this purpose. We took it that nothing was known about the observers, so we considered them both as *c*-observers ($\alpha = 100$). The reduction, then, consists in subtracting the quantity $B(c, w) - B(c, f)$ (Table II) from the observed magnitudes.

DR's observations being more numerous, we have used his curve to yield the magnitudes for the dates of Bl's observations. The result is given in Table IV. It shows that, whereas the differences between the original WL-curves always ran in the same direction and had a mean value of $0^m.54$, the final "normal-eye"-curves based on observations with *one* filter only differ by $0^m.16$, *plus* and *minus* signs alternating.*

Table IV

Julian Date	DR (curve)	Bl (direct)	DR – Bl	
			Filter	White Light
	m	m	m	m
6391	8·72	8·13	+0·59	+0·75
6405	8·18	7·99	+ ·19	+0·81
6413	7·68	7·69	− ·01	+0·56
6420	7·20	7·38	− ·18	+1·09
6432	6·20	6·00	+ ·20	+0·44
6467	5·72	5·81	− ·09	0·00
6489	6·38	5·95	+ ·43	+0·16

* This mean difference would be even smaller if the last number could be left out. This was obtained after a rather dangerous extrapolation of DR's curve ; moreover, the last observations had to be made in the evening twilight.

From this it will be seen that if it were desirable to restrict the observations to a single filter, such observations would already lead to a far better agreement between various observers. Once the transmission-curve of a filter is known, the quantities $B(c, w) - B(c, f)$ can be readily computed. Only a rough knowledge of the star's temperature is wanted for this calculation ; as may be learned from Table II, a change of about 400° in the assumed value of T leads to a difference of only $0^{m}.1$ in the brightness for a "normal-eye."

9. *Two Criteria for a good Colour-filter.*—If, as we believe to have made plausible, the cause of systematic differences between the results of various observers, estimating the brightness of a red variable star, lies in the different retina-activity of these observers, we can accurately tell the two conditions which a filter will have to fulfil in order to reduce such differences to a minimum. We shall call them the "characteristic numbers," I and II, of the filter.

I. The first, and chief, requirement of a colour-filter is fulfilled when two observers, using it, will see the same differences of brightness between stars of various spectral classes, whether they are *c*- or *r*-observers. If this requirement is not entirely fulfilled (and, in practice, it probably never will be), the difference between a *c*- and an *r*-observer will be largest when they compare a red star with an Ao star (comparison stars of class B are a rare exception).

Since the energy-curves of M8 to S stars are unknown, we shall call a "red" star a black body at 2000°. The first characteristic quantity of a filter then becomes

$$\mathrm{I} = [B(c, f) - B(r, f)]_{2000^\circ} - [B(c, f) - B(r, f)]_{\mathrm{Ao}}.$$

We may repeat that I represents the greatest possible divergence between a 100 per cent. *c*-observer and a 100 per cent. *r*-observer, when both compare the brightness of a red star with that of a star of spectral class Ao.

II. The first requirement will be better fulfilled if the filter is more closely monochromatic. Very monochromatic filters, however, are always dark ; observations through them would be very difficult and, in many telescopes, would not allow observation of the star's minimum. This means that a good colour-filter will have to fulfil a second requirement, viz. it should reduce the star's brightness as little as possible. We have called this reduction, peculiar to a certain filter, its "characteristic number" II.

For our filters the quantities II have already been given in Table II. Since we are exclusively concerned with the variable star, only the lowest line in this table ($T = 2000^\circ$) will count. Shall we take the numbers on this line for a *c*- or for an *r*-observer ? We decide for the *r*-observer, because our second criterion is most important at the star's minimum brightness, where most observers may be supposed to use their rods. The second "characteristic number" of a filter, then, becomes

$$\mathrm{II} = [B(r, w) - B(r, f)]_{2000^\circ}.$$

Table V contains the characteristic numbers I and II for the five filters

considered in this paper, and, in addition, for a few filters, occurring with their transmission-curves in the catalogue of the Schott Glass Works, Jena. The numbers I and II have been taken as abscissæ and ordinates in a graphical representation of filter-qualities (see fig. 5). On the line of abscissæ the point at $0^m.88$ is the "no-filter"-point; it represents the characteristic number I for white light.

TABLE V

Filter	I	II	Filter	I	II
	m	m		m	m
Bf	0·04	6·09	Schott BG 7	0·58	0·76
F_1	0·41	1·48	,, GG 11 (2 mm.)	0·71	0·25
F_8	0·71	0·42	,, VG 1 (1 mm.)	0·59	0·49
F_{13}	0·51	1·14	,, ,, (2 mm.)	0·42	0·84
Rf	0·43	1·94	White light	0·88	0·00

In this diagram the "ideal filter" is represented by the origin, I = 0; II = 0.

We may learn the following from this graphical representation:

(1) The filters GG 11, F_{13} and Rf are "one-sided," *i.e.* they transmit nothing from the violet to a certain λ, and from there to the red practically 100 per cent. Such filters are filed along a smooth curve in our diagram; near their representative points, *g*, *d* and *e*, the transmission limits have been added in brackets. The characteristic numbers of filters of the same group may be interpolated along this curve, instead of being computed.

(2) Schott's yellow filter GG 11 has been recommended to us for work on variable stars. It absorbs only the violet, the blue and the beginning of the green up to λ 5000 A. This filter has the advantage of a small number II, and work with it will not cause any difficulty. But since its number I is $0^m.71$, that for white light being $0^m.88$, it will not be able to fulfil its principal task, viz. to bring the estimates of observers with different retina-activities into good agreement. This result may be compared with Escher Desrivière's statement (*loc. cit.*, p. 399), that extinction of the blue and the violet does not lead to any noteworthy decrease of the observed brightness of a white luminous point.

(3) Apparently, the green-blue filter VG 1 (thickness 2 mm.) by Schott is much to be preferred to either of the one-sided filters. So long as no better filter is found, we are inclined to recommend this filter for coloured light observations. It makes a star only $0^m.8$ fainter to a *c*-observer, $1^m.4$ to an *r*-observer, and reduces the Purkinje-difference from $0^m.88$ to $0^m.41$.

10. *Appendix.*—At our request Professor Dr. H. J. M. Weve, Director of the Ophthalmological Institute of the University at Utrecht, had the kindness personally to submit DR and Bl to a colour-vision test. The result was that Bl's colour-vision proved to be practically normal. On the contrary, DR's sensitivity for green light was far below normal; he proved

to be extremely deuteranomalous. The influence of this on the sensitivity curve c_λ is much less than might be anticipated. Measurements by Coblentz and Emerson (*Bulletin Bureau of Standards*, 14, 196, 1918, figs. 5 and 6) show that the sensitivity curve of deuteranomalous, and even of colour-blind people, is displaced to the red nowhere more than 100 A. Since the displacement, rods *minus* cones, amounts to 300 to 600 A., anomalous colour-vision is insignificant compared therewith.

The consequences of this state of affairs may be summed up as follows :

(1) The abnormal colour-sensitivity of DR would cause him to estimate

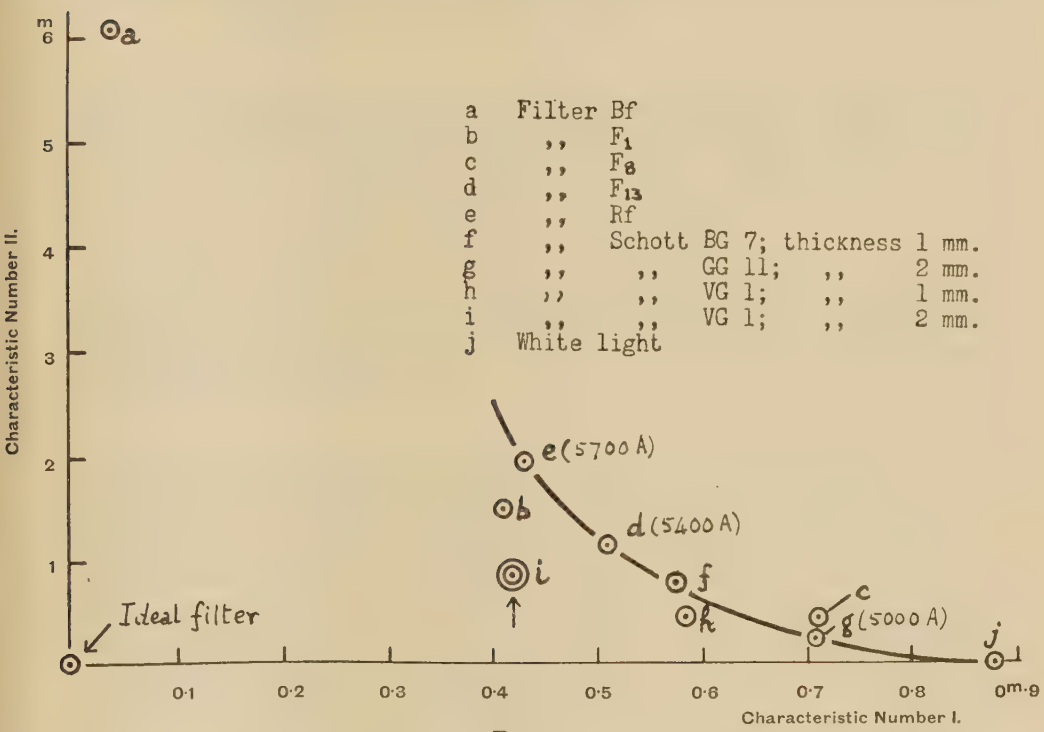


FIG. 5.

a red star *brighter* than Bl. Since the reverse happens to be the case, our hypothesis that the action of cones and rods is chiefly responsible seems the more probable.

(2) Since DR is chiefly an *r*-observer, the peculiarities of his colour-vision will hardly influence his observations.

(3) But, even if DR's peculiar colour-vision does slightly affect his WL-observations, it will have hardly any influence on the "normal-eye"-curve derived from filter-observations. The agreement, shown in Table IV, proves this. It thus becomes clear that the observations in coloured light will, apart from the advantages already mentioned, lead to a better agreement between observers with very different colour-sensitivities.

(4) The value of the temperature of R Leo ($T = 2350°$), derived from Bl's observations, must be nearer the truth than that of DR.

11. *A few Desiderata.*—(1) An observer of red stars should have the colour-vision of his eyes tested.

(2) The reduction of filter-observations of variable stars requires a knowledge of the spectra of all the comparison stars. A rough estimation of the colour-indices would do. Colour-indices of comparison stars for red variables in Hagen's A.S.V. would be very welcome.

(3) Photographic spectra of variable stars of the classes M8 – S on panchromatic plates are urgently needed, in order to give the distribution of energy in the visible part of the spectrum.

OBSERVATIONS OF SOLAR FLOCCULI MADE WITH THE SPECTROHELIOSCOPE AT THE ROYAL OBSERVATORY, GREENWICH, DURING THE YEAR 1931.

(Communicated by the Astronomer Royal)

This note summarises the observations, made with the spectrohelioscope during the year 1931, of bright and dark flocculi seen on the Sun's disc in *Hα* light. Observations were commenced regularly with this instrument in 1930, and a summary for that year is given in *Monthly Notices*, **91**, 593, 1931.

The spectrohelioscope, which has been lent to Greenwich by the Mount Wilson Observatory, is used in conjunction with a Rowland grating giving a first-order spectrum, the scale being approximately 1 mm. = 4 A. The width of the second slit of the instrument was usually 0·1 mm., *i.e.* about one-third of the width of the *Hα* line in the first-order spectrum which was used for observation throughout the year. The diameter of the monochromatic image of the Sun's disc at the second slit is about 50 mm., of which a strip 6 mm. wide and 28 mm. long is seen through the eyepiece when the pair of rectangular prisms are rotated by an electric motor. The instrument is fully described by Dr. Hale in the *Astrophysical Journal*, **70**, 265–311, 1929.

Measures of radial velocity are made with the "line-shifter," the scale of which is from 0–10 divisions = 0·37 A. = 17 km./sec. at *Hα*. The zero of the *Hα* line is determined from measures of the darkest part of the line from an undisturbed portion of the Sun near the centre of the disc. All measured displacements are interpreted as being due to the effect of radial velocity. The measures have not been corrected for the effect of the Sun's rotation or of the Earth's orbital and rotational velocity.

The probable error of a single measure of radial velocity, as determined from a number of successive measures, is about 3 km./sec., including the probable error of the zero determination. Four measures being usually made on each flocculus, the probable errors of the tabulated values are of the order of 2 km./sec.

Observations were made on 134 days during the year 1931 ; on about 81 days the survey of the Sun's disc was complete or nearly so. In the

course of searching for active areas on the Sun, measures of radial velocity were made systematically on all flocculi, both bright and dark. Summaries of these measures are given in the following tables, which may be compared with those given for the year 1930 in the summary already referred to. The observations have been classified as follows :—

(*a*) Areas of bright *Hα* flocculi.
(*b*) Dark *Hα* flocculi not associated with sunspots or bright areas.
(*c*) Dark *Hα* flocculi associated with sunspots.
(*d*) Dark *Hα* flocculi associated with bright areas not containing sunspots on the day of observation.
(*e*) Unusually bright *Hα* flocculi.

(*a*) Measures were made of 111 areas of bright flocculi in the central part of the Sun's disc, contained by a circle of diameter half that of the disc. The average measured displacement of the wave-length at the position of their maximum intensity from the centre of the *Hα* line is equivalent to a radial velocity of −1·1 km./sec. Table I gives the frequency of the measured values ranging from −10 to +5 km./sec., a minus sign indicating an outward motion from the Sun and a plus sign an inward motion. For purposes of comparison, measures of 138 bright flocculi outside the central part of the disc are also tabulated, the mean average displacement of these being equivalent to −0·7 km./sec.

TABLE I

Measures of Radial Velocity of Bright Hα Flocculi, 1931

Velocity km./sec.	Number		Velocity km./sec.	Number	
	Central	Non-central		Central	Non-central
−10	1	1	+1	15	16
9	0	0	2	8	11
8	0	0	3	3	6
7	4	3	4	1	2
6	2	4	5	2	3
5	4	5	6	0	1
4	6	7	7	0	1
3	14	15	8	0	0
2	13	20	9	0	0
−1	19	15	10	0	1
0	19	26	+11	0	1
			Means	−1·1 km./sec.	−0·7 km./sec.

Table I shows that the bright *Hα* flocculi are not associated with large vertical movements in the sense usually applicable to chromospheric phenomena. The distribution of the velocities corresponds essentially to that of a probable error of 2 km./sec. Whether any significance can be attached to the mean outward motion of 1·1 km./sec. for the central flocculi cannot be said without measures with higher dispersion.

(*b*) Measures were made of 92 dark flocculi not associated with sunspots or bright *Hα* flocculi. These were usually conspicuous dark markings a

few minutes of arc in length and tending to avoid sunspots. It was characteristic of them that, as the line-shifter was turned, they appeared or disappeared as a whole, thus exhibiting no appreciable progression of radial velocity along their length. In duration they approximated to that of sunspots. During 1931 the dark flocculi of this class werel ess numerous and extensive than in 1930.

The mean of the measured radial velocities of these 92 flocculi was −1·0 km./sec. Table II summarises the measures both for these and for the flocculi in the non-central part of the Sun's disc, the latter numbering 241 and giving a mean measured radial velocity of +0·4 km./sec.

TABLE II

Measures of Radial Velocity of Dark Hα Flocculi not associated with Sunspots or Bright Flocculi, 1931

Velocity km./sec.	Number		Velocity km./sec.	Number	
	Central	Non-central		Central	Non-central
−35	0	1	− 1	5	36
...			0	19	37
−23	1	1	+ 1	14	27
...			2	10	30
−14	1	0	3	8	22
10	1	1	4	4	12
9	1	2	5	0	10
8	1	1	6	0	4
7	0	3	7	0	2
6	1	4	8	0	2
5	4	2	...		
4	4	4	22	0	1
3	5	16	...		
− 2	13	22	+27	0	1
			Means	−1·0 km./sec.	+0·4 km./sec.

It may be concluded from Table II that the dark *Hα* flocculi not associated with sunspots or bright areas have, in general, scarcely measurable radial movements. The few large velocities tabulated are, however, doubtless real, and appear to represent statistically a rather infrequent and brief eruptive phase of some of these dark flocculi. Their behaviour is in marked contrast to that of flocculi associated with sunspots and summarised in Table III.

(*c*) Table III gives a tabulation of measured radial velocities of 143 dark flocculi that were observed in the immediate neighbourhood of sunspots to which many of them, judging from the character of their motion, were evidently related. The flocculi of this class were generally short-lived (those with the larger velocities lasting for half an hour or less) and were not so extensive as those of class (*b*), the average length being a little less than 1′. Several of these flocculi were small, dense, circular markings about 20″ in diameter. Progression of radial velocity along the length of the filamentary

markings was observed in about 30 per cent. of the cases. For the purpose of this tabulation the largest measured value of such observed progression of velocities has been taken in each case irrespective of sign. The detailed measures will be published in the *Greenwich Photoheliographic Results* for 1931, in continuation of those already published in the volume for 1930.

TABLE III

Measures of Radial Velocity of Dark Hα Flocculi associated with Sunspots, 1931

Velocity km./sec.	Number	Velocity km./sec.	Number
0–4	29	40–49	15
5–9	6	50–59	7
10–19	8	60–69	3
20–29	30	70–79	2
30–39	42	95	1

An inspection of Table III shows the active nature of a large proportion of the dark flocculi observed in association with sunspots. A maximum occurs between velocities of 20–50 km./sec., and it will be of interest to see whether this bears any relation to the 11-year cycle. The signs of the velocities in Table III show an excess of inward motions.

(*d*) The activity of dark flocculi referred to in Table III seems to depend to a large degree upon the presence of sunspots, however small, and not usually upon the presence of bright flocculi alone, which are invariable accompaniments of sunspots. This fact is suggested by Table IV, which gives the frequency of radial velocities of dark *Hα* flocculi observed during 1931 associated with areas of bright *Hα* flocculi, but in which no sunspot was recorded at Greenwich at or about the time of observation of the flocculus.

TABLE IV

Measures of Dark Flocculi associated with Bright Flocculi but not with Sunspots, 1931

Velocity (km./sec.)	0–4	5–9	10–19	20–29	30–39	70 *
Number	29	7	1	4	5	1

Although in Table IV there are a few velocities between 20 and 40 km./sec., the numbers are small compared with those in Table III.

(*e*) Four small eruptions of bright *Hα* flocculi were recorded during 1931,

* This large value, comprising velocities from +50 to −70 km./sec., was observed on 1931 November 11 between $11^h\ 4^m$ and $11^h\ 39^m$ at the approximate heliographic position of longitude 155° (17° east of the central meridian), latitude 3° south, associated with a very small but intensely dark flocculus adjacent to a tiny bright flocculus. No sunspot is visible on the solar photographs taken at Greenwich 2 hours earlier ; it would be of interest to know whether a small sunspot was recorded in the above position by any other observatory, and whether a magnetic field was present, denoting an " invisible sunspot."

and are tabulated below, in continuation of a similar table given for 1930 in *Monthly Notices*, **91**, 600, 1931. The unit of area in column 4 is one millionth of the Sun's hemisphere.

TABLE V

Eruptions of Bright Hα Flocculi, 1931

Date of Eruption	Associated Sunspot			Magnetic Storm Commenced
	C.M.P.	Lat.	Area	
Mar. 17·5	Mar. 11·9 *	4° S.	135	Mar. 12·7
Apr. 12·5	Apr. 16·5	7° N.	250	None
July 12·7	July 10·5	7° S.	50	None
Nov. 6·5	Nov. 7·6	0°	100	None

The eruptions on July 12 and November 6 illustrate the abruptness with which the phenomenon may occur, the regions in question having been under observation 13 minutes and 2 minutes respectively before the eruptions were first seen to be in progress. These have no appreciable radial velocity.

Royal Observatory, Greenwich :
1932 February 5.

ON MOTION IN THE NEIGHBOURHOOD OF THE EQUILATERAL POINTS OF LIBRATION.

H. C. Plummer, F.R.S.

The extensive researches on the restricted problem of three bodies prosecuted at the Copenhagen Observatory have shown very clearly the importance of the five Lagrangean centres of libration. They have also brought out the important part played by asymptotic solutions in addition to the periodic orbits first investigated in a general manner by Sir George Darwin. A few notes suggested by recent work at Copenhagen and elsewhere are now given on the infinitesimal motions in the immediate neighbourhood of the equilateral points of libration for the whole range of possible mass-ratios.

1. In a field rotating with unit angular velocity, the relative motion of a small mass in the neighbourhood of a point (x_0, y_0) is given by the equations

$$\ddot{x} - 2\dot{y} = \Omega_{10} + \Omega_{20}x + \Omega_{11}y + \cdots$$
$$\ddot{y} + 2\dot{x} = \Omega_{01} + \Omega_{11}x + \Omega_{02}y + \cdots$$

where Ω is a potential function and

$$\Omega_{ij} = \frac{\partial^{i+j}\Omega}{\partial^i x_0 \partial^j y_0}, \qquad v^2 = 2\Omega - C.$$

* Sunspots first visible on March 17 at a distance of more than 70° west of the central meridian.

When (x_0, y_0) is a point of libration, $\Omega_{10} = \Omega_{01} = 0$, and to the lowest order

$$\left.\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_{20}x + \Omega_{11}y \\ \ddot{y} + 2\dot{x} &= \Omega_{11}x + \Omega_{02}y \end{aligned}\right\}, \tag{1}$$

from which, after eliminating $\dot{y}$, $\ddot{y}$,

$$(4\Omega_{02} - \Omega_{11}^2)y = 2\dddot{x} - \Omega_{11}\ddot{x} + 2(4 - \Omega_{20})\dot{x} - (4 - \Omega_{20})\Omega_{11}x. \tag{2}$$

The period equation, giving any possible period $2\pi/m$, is

$$m^4 - m^2(4 - \Omega_{20} - \Omega_{02}) + \Omega_{20}\Omega_{02} - \Omega_{11}^2 = 0. \tag{3}$$

The roots being m_1^2, m_2^2, both m_1 and m_2 may be taken to be positive when real.

2. In the restricted problem of three bodies,

$$2\Omega = \mu\left(\frac{2}{r_1} + r_1^2\right) + \mu'\left(\frac{2}{r_2} + r_2^2\right),$$

where $\mu + \mu' = 1$ with suitable units. At the equilateral points of libration, $r_1 = r_2 = 1$, $\Omega_{10} = \Omega_{01} = 0$, and *

$$\left.\begin{aligned} \Omega_{20} = \tfrac{3}{4}, \qquad \Omega_{11} = \pm\frac{3\sqrt{3}}{4}(\mu - \mu'), \qquad \Omega_{02} = \tfrac{9}{4}, \\ \Omega_{20}\Omega_{02} - \Omega_{11}^2 = \tfrac{27}{4}\mu\mu'. \end{aligned}\right\} \tag{4}$$

At these points therefore 2Ω is a minimum with the value 3 ; at no point is Ω a maximum.

Let ρ_1, ρ_2 be the principal radii of curvature of the surface $z = \Omega$ at a minimum point. They are the roots of the quadratic

$$\rho^2(\Omega_{20}\Omega_{02} - \Omega_{11}^2) + \rho(\Omega_{20} + \Omega_{02}) + 1 = 0$$

and are negative, being drawn in the upward direction. If then

$$\begin{aligned} \rho_1\nu = \rho_2\lambda &= -1, \qquad \nu > \lambda > 0, \\ \nu + \lambda &= \Omega_{20} + \Omega_{02}, \qquad \nu\lambda = \Omega_{20}\Omega_{02} - \Omega_{11}^2. \end{aligned} \tag{5}$$

By comparison with (3),

$$m_1^2 + m_2^2 = 4 - \nu - \lambda, \qquad m_1^2 m_2^2 = \lambda\nu, \tag{6}$$

whence

$$(\sqrt{\nu} - \sqrt{\lambda})^2 = 4 - m_1^2 - m_2^2 - 2m_1m_2 = 4 - (m_1 + m_2)^2,$$

and it follows that

$$m_1 + m_2 < 2; \tag{7}$$

or if T_1, T_2 are the periods in the infinitesimal orbits and T is the rotation period of the field,

$$\frac{1}{T_1} + \frac{1}{T_2} < \frac{2}{T}.$$

3. The introduction of the curvatures ν, λ in the place of the radii of curvature of the surface $z = \Omega$ is all that is necessary to connect a former

* *Dynamical Astronomy*, p. 243.

paper of the writer * with recent work † by Mr. M. Martin and by Miss Rosenthal. Thus the condition for the existence of these infinitesimal orbits at a point where Ω is a minimum may be written

$$\sqrt{\nu}+\sqrt{\lambda}<2. \tag{8}$$

Contour lines defined by $z=\frac{1}{2}C$ are the curves of zero velocity $2\Omega=C$, and in the immediate neighbourhood of a minimum point these become the ellipses

$$\Omega_{20}x^2+2\Omega_{11}xy+\Omega_{02}y^2=C-C_0, \tag{9}$$

provided that $C>C_0$, where C_0 is the value of C at the minimum point. Without this condition no such curve exists.

Now the equation of an orbit is ‡

$$(\Omega_{20}+m^2)x^2+2\Omega_{11}xy+(\Omega_{02}+m^2)y^2=\frac{4(C-C_0)}{4-\Omega_{20}-\Omega_{02}-2m^2}.$$

But, by (3), when the two orbits exist,

$$m_1^2+m_2^2=4-\Omega_{20}-\Omega_{02}.$$

The two orbits are therefore

$$(\Omega_{20}+m_1^2)x^2+2\Omega_{11}xy+(\Omega_{02}+m_1^2)y^2=\frac{4(C-C_0)}{m_2^2-m_1^2}, \tag{10}$$

$$(\Omega_{20}+m_2^2)x^2+2\Omega_{11}xy+(\Omega_{02}+m_2^2)y^2=\frac{4(C-C_0)}{m_1^2-m_2^2}. \tag{11}$$

It is immediately evident that (9) and (10) represent real curves if $C>C_0$ and $m_2>m_1$, while, with $m_2>m_1$, (11) represents a real curve only when $C<C_0$ and (9) does not exist.

4. Let a, b be the semi-axes of the orbit (10) and α, β the semi-axes of the corresponding curve (9) of zero velocity. Then

$$\begin{aligned}\frac{1}{a^2b^2}&=\{(\Omega_{20}+m_1^2)(\Omega_{02}+m_1^2)-\Omega_{11}^2\}(m_2^2-m_1^2)^2/16(C-C_0)^2\\&=m_1^2(m_2^2-m_1^2)^2/4(C-C_0)^2\end{aligned}$$

by (3). Similarly

$$\begin{aligned}\frac{1}{\alpha^2\beta^2}&=(\Omega_{20}\Omega_{02}-\Omega_{11}^2)/(C-C_0)^2\\&=m_1^2m_2^2/(C-C_0)^2.\end{aligned}$$

Hence, by (7),

$$\frac{\alpha\beta}{ab}=\frac{m_2^2-m_1^2}{2m_2}<\frac{m_2-m_1}{m_2}<1.$$

In the nature of things the orbit cannot meet the corresponding curve of zero velocity, which therefore lies wholly inside the orbit since it has the

* *M.N.*, **62**, 6, 1901.

† *A.N.*, 5840. In the second paper (column 171) the third and fourth columns of the table under L_3 are evidently transposed.

‡ *M.N.*, **62**, 12, 1901.

smaller area. This fact, noted by Martin in the restricted problem, is thus general for a point where Ω is a minimum.

5. The coincidence of the major axes of the ellipses (9), (10) and (11) was proved generally for a point where Ω is a minimum. The same property does not extend to both the orbits at a point where Ω is a maximum, but this case does not arise in the restricted problem.

The eccentricity of an orbit is given * by

$$e^2 = \frac{\rho_2^{-1} - \rho_1^{-1}}{m^2 - \rho_1^{-1}} = \frac{\nu - \lambda}{\nu + m^2},$$

or

$$\frac{e^2}{2 - e^2} = \frac{\nu - \lambda}{\nu + \lambda + 2m^2}. \tag{12}$$

But (3) becomes, by (4),

$$m^4 - m^2 + \tfrac{27}{4}\mu\mu' = 0, \tag{13}$$

and therefore, by (5),

$$\frac{e^2}{2 - e^2} = \frac{\sqrt{(9 - 27\mu\mu')}}{4 \pm \sqrt{(1 - 27\mu\mu')}}.$$

Thus the eccentricities of the two orbits are given by

$$e = \sqrt{2}\,.\,\sin\epsilon, \qquad \tan^2\epsilon = \frac{3\sqrt{(1 - 3\mu\mu')}}{4 \pm \sqrt{(1 - 27\mu\mu')}}, \tag{14}$$

when $27\mu\mu' < 1$. If $m_2 > m_1$, $e_2 < e_1$. Numerical values are given in Table I.

TABLE I

μ	$(C > 3)$ e_1	$(C < 3)$ e_2
0·000	1·000000	0·866025
·004	0·993913	·867410
·008	·987625	·868984
·012	·981095	·870792
·016	·974268	·872884
·020	·967055	·875358
·024	·959320	·878344
·028	·950824	·882084
·032	·941060	·887086
0·036	0·928417	0·894956
$(27\mu\mu' = 1)$	0·910180	

6. When Ω is a minimum, but the period equation (3) has complex roots, let

$$im = n(\cos\phi + i\sin\phi). \tag{15}$$

Then (3) gives

$$n^4\cos 4\phi + n^2\cos 2\phi(4 - \Omega_{20} - \Omega_{02}) + \Omega_{20}\Omega_{02} - \Omega_{11}^2 = 0,$$
$$n^4\sin 4\phi + n^2\sin 2\phi(4 - \Omega_{20} - \Omega_{02}) = 0;$$

* *M.N.*, **62**, 11, 1901.

or

$$n^2 \cos 2\phi = -\tfrac{1}{2}(4 - \Omega_{20} - \Omega_{02}), \tag{16}$$

$$n^4 = \Omega_{20}\Omega_{02} - \Omega_{11}^2. \tag{17}$$

By the use of (2) a solution of (1) can be written

$$x = A(4\Omega_{02} - \Omega_{11}^2)e^{nt\cos\phi} \sin(nt \sin\phi), \tag{18}$$

$$y = Ae^{nt\cos\phi}\{2n^3 \sin(nt\sin\phi + 3\phi) - n^2\Omega_{11}\sin(nt\sin\phi + 2\phi) + 2n(4 - \Omega_{20})\sin(nt\sin\phi + \phi) - (4 - \Omega_{20})\Omega_{11}\sin(nt\sin\phi)\},$$

or

$$y = Ae^{nt\cos\phi}\{k_1 \sin(nt\sin\phi) + k_2\cos(nt\sin\phi)\}, \tag{19}$$

where

$$k_1 = 2n^3\cos 3\phi - n^2\Omega_{11}\cos 2\phi + 2n(4 - \Omega_{20})\cos\phi - (4 - \Omega_{20})\Omega_{11},$$
$$k_2 = 2n^3\sin 3\phi - n^2\Omega_{11}\sin 2\phi + 2n(4 - \Omega_{20})\sin\phi.$$

When $t = 0$, $x = 0$, $y = Ak_2$ and

$$\dot{x} = An(4\Omega_{02} - \Omega_{11}^2)\sin\phi,$$
$$\dot{y} = An(k_1 \sin\phi + k_2\cos\phi)$$
$$= An\{2n^3\sin 4\phi - n^2\Omega_{11}\sin 3\phi + 2n(4 - \Omega_{20})\sin 2\phi - (4 - \Omega_{20})\Omega_{11}\sin\phi\}.$$

Therefore at this instant

$$\frac{\dot{y}}{\dot{x}} = \frac{8n^3\cos\phi\cos 2\phi - n^2\Omega_{11}(2\cos 2\phi + 1) + 4n(4 - \Omega_{20})\cos\phi - (4 - \Omega_{20})\Omega_{11}}{4\Omega_{02} - \Omega_{11}^2}$$
$$= \frac{4n\cos\phi \,.\, \Omega_{02} - \Omega_{11}(n^2 + \Omega_{02})}{4\Omega_{02} - \Omega_{11}^2}.$$

The inclination of the spiral to the axis of x (line of syzygies) is represented by the damping factor $n\cos\phi$ when $\Omega_{11} = 0$, which is the case when $\mu = \mu'$ (Strömgren); or more generally, when the axis of x is parallel to a line of curvature at the minimum point on the surface $z = \Omega$.

7. With the simplification introduced by the latter choice of axes,

$$\Omega_{11} = 0, \qquad \Omega_{20} = \lambda, \qquad \Omega_{02} = \nu,$$

and (16), (17) become

$$n^4 = \lambda\nu, \qquad n^2\cos 2\phi = -\tfrac{1}{2}(4 - \lambda - \nu). \tag{20}$$

The motion considered is now given by

$$\left.\begin{aligned} x &= 4A\nu \,.\, e^{nt\cos\phi}\sin(nt\sin\phi) \\ y &= Ae^{nt\cos\phi}\{k_1\sin(nt\sin\phi) + k_2\cos(nt\sin\phi)\} \end{aligned}\right\}, \tag{21}$$

where

$$k_1 = 2n^3\cos 3\phi + 2n(4 - \lambda)\cos\phi,$$
$$k_2 = 2n^3\sin 3\phi + 2n(4 - \lambda)\sin\phi;$$

or

$$\left.\begin{aligned} k_1 &= 2n\cos\phi \,.\, \sqrt{\nu}(\sqrt{\nu} - \sqrt{\lambda}) \\ k_2 &= 2n\sin\phi \,.\, \sqrt{\nu}(\sqrt{\nu} + \sqrt{\lambda}) \end{aligned}\right\}. \tag{22}$$

The result of eliminating the periodic terms from the motion is

$$16A^2k_2^2\nu^2e^{2nt\cos\phi} = k_2^2x^2 + (4\nu y - k_1x)^2$$
$$= 8\nu(2x^2\sqrt{\nu\lambda} - k_1xy + 2\nu y^2),$$

for it is easily found that

$$k_1^2 + k_2^2 = 16\nu\sqrt{\nu\lambda}.$$

The spiral motion may therefore be expressed as an exponential expansion of the ellipse ($0° < \phi < 90°$)

$$x^2\sqrt{\lambda} - n\cos\phi(\sqrt{\nu} - \sqrt{\lambda})\,.\,xy + y^2\sqrt{\nu} = 1, \tag{23}$$

or the asymptotic contraction of the ellipse

$$x^2\sqrt{\lambda} + n\cos\phi(\sqrt{\nu} - \sqrt{\lambda})xy + y^2\sqrt{\nu} = 1, \tag{24}$$

resulting from changing the sign of $\cos\phi$. In either case the eccentricity is given by

$$\frac{1-e^2}{(2-e^2)^2} = \frac{1}{a^2b^2(a^{-2}+b^{-2})^2} = \frac{4\sqrt{\lambda\nu} - n^2\cos^2\phi(\sqrt{\nu}-\sqrt{\lambda})^2}{4(\sqrt{\lambda}+\sqrt{\nu})^2}$$
$$= \frac{16\sqrt{\lambda\nu} - (\sqrt{\nu}-\sqrt{\lambda})^2(2\sqrt{\lambda\nu} - 4 + \lambda + \nu)}{16(\sqrt{\lambda}+\sqrt{\nu})^2}$$
$$= \tfrac{1}{16}\{4 - (\sqrt{\nu}-\sqrt{\lambda})^2\},$$
$$\frac{e^2}{2-e^2} = \tfrac{1}{2}(\sqrt{\nu}-\sqrt{\lambda})$$
$$= \tfrac{1}{2}\{\Omega_{20} + \Omega_{02} - 2(\Omega_{20}\Omega_{02} - \Omega_{11}^2)^{\frac{1}{2}}\}^{\frac{1}{2}}$$

in the original notation. In the present case, by (4),

$$e = \sqrt{2}\,.\,\sin\epsilon, \qquad \tan^4\epsilon = \tfrac{3}{4}(1 - \sqrt{3\mu\mu'}). \tag{25}$$

At the same time the combination of (4) with (16) and (17) gives

$$\kappa^2 = n^2\cos^2\phi = \tfrac{1}{4}[3\sqrt{3\mu\mu'} - 1],$$
$$\omega^2 = n^2\sin^2\phi = \tfrac{1}{4}[3\sqrt{3\mu\mu'} + 1].$$

The eccentricities of the generating ellipses (23), (24) are given numerically in Table II.

TABLE II

μ	e	μ	e
0·050	0·901026	0·300	0·751390
·100	·866138	·350	·728425
·150	·834872	·400	·710050
·200	·805377	·450	·698020
0·250	0·777392	0·500	0·693818

8. When the period equation (3) has repeated roots, given by

$$2m^2 = 4 - \Omega_{20} - \Omega_{02}, \tag{26}$$

the two families of elliptic orbits coincide before complex roots give rise to asymptotic solutions. An alternative solution to the one elliptic family can then be expressed by a motion through the point of libration with assigned velocity. If at $t=0$, $x=y=0$ and $\dot{x}=U$, $\dot{y}=V$,

$$x=A_1 t\cos mt+A_2 t\sin mt+(U-A_1)m^{-1}\sin mt$$
$$y=B_1 t\cos mt+B_2 t\sin mt+(V-B_1)m^{-1}\sin mt$$

are appropriate forms for insertion in (1). The result of comparing terms in $\cos mt$ is

$$mA_2=V,\qquad mB_2=-U,$$

while the comparison of terms in $t\sin mt$ gives

$$2m^2B_1=(\Omega_{20}+m^2)V-\Omega_{11}U,$$
$$2m^2A_1=(\Omega_{02}+m^2)U-\Omega_{11}V.$$

Similar comparisons yield two other pairs of equations, but these reduce to the period equation (3) with the condition for repeated roots expressed by (26).

Thus the secular solution takes the form

$$2m^3x=mt\cos mt\{(\Omega_{02}+m^2)U-\Omega_{11}V\}+2m^2Vt\sin mt$$
$$+\sin mt\{(m^2-\Omega_{02})U+\Omega_{11}V\},$$

$$2m^3y=mt\cos mt\{(\Omega_{20}+m^2)V-\Omega_{11}U\}-2m^2Ut\sin mt$$
$$+\sin mt\{(m^2-\Omega_{20})V+\Omega_{11}U\}.$$

In addition to the velocity components U and V, two independent epochs can be assigned to t according as it occurs inside or outside the circular functions. The solution is then made completely general.

The direction of curvature at the origin is shown by the circle of curvature,

$$x^2+y^2-Vx+Uy=0.$$

In the case considered, $m^2=\frac{1}{2}$, $4\Omega_{11}=\sqrt{23}$ and

$$x=\frac{t}{4}\cos\frac{t}{\sqrt{2}}(11U-\sqrt{23}V)+\sqrt{2}Vt\sin\frac{t}{\sqrt{2}}-\frac{1}{2\sqrt{2}}\sin\frac{t}{\sqrt{2}}(7U-\sqrt{23}V),$$

$$y=\frac{t}{4}\cos\frac{t}{\sqrt{2}}(5V-\sqrt{23}U)-\sqrt{2}Ut\sin\frac{t}{\sqrt{2}}-\frac{1}{2\sqrt{2}}\sin\frac{t}{\sqrt{2}}(V-\sqrt{23}U),$$

is the above solution, while

$$5x^2+2\sqrt{23}xy+11y^2=c^2$$

represents the single family of elliptic orbits. The axes of the latter are in the ratio $\sqrt{2}+1:1$.

A BRIGHT METEOR PASSING THE PLEIADES.

William J. S. Lockyer, M.A., Ph.D., Major (late R.A.F.). (Plate 4.)

Quite recently a series of four Zeiss triplet lenses has been equatorially mounted and housed at the Norman Lockyer Observatory through the generosity of Dr. Robert L. Mond, the construction having been undertaken by Messrs. Cooke, Troughton & Simms, Ltd.

During the months of November, December and January last the instrument was in use to secure the adjustment of the mounting, to rate the driving clock and control pendulum, and to determine the focal planes of the cameras.

On the night of January 11, when all the above adjustments had been approximately determined and the driving clock, with which there had been a little difficulty, seemed to be going well, a backed Ilford "Iso-Zenith" plate was exposed at $21^h\ 40^m$ U.T. in the Zeiss triplet of focal length 70 cm. ($f/5$) to the region of the Pleiades. After running for about a quarter of an hour, the star image in the finder momentarily trailed somewhat owing to some irregularity in the clock. The star image was immediately reset on the cross-wires in the finder, and after that the clock performed splendidly.

At $23^h\ 10^m$ another Zeiss triplet camera, loaded with a similar kind of plate, was brought into action : this had a focal length of 50 cm. ($f/4{\cdot}8$). Both cameras after this were run without a hitch until $2^h\ 40^m$ U.T. when they were closed. The night was cloudless, but the seeing was never very good.

Both plates on development showed a meteor trail, which entered at one end of the plates but faded out before it reached the opposite sides of the plates. The greatest luminosity of the trail occurred at that portion nearest the Pleiades. The length of the trail on the plate of the 50 cm. camera covers 12°, while that portion where the largest amount of volatilisation occurred is about 3° in length.

The fact that both cameras recorded it indicated that its apparition must have occurred between $23^h\ 10^m$ and $2^h\ 40^m$ U.T. Reproductions of the meteor photographed on the two plates are shown in Plate 4, figs. 1 and 2 : they are magnified a little over two diameters.

Fig. 1 illustrates that taken with the triplet of 50 cm. focal length. In this case the driving clock ran well for the whole time of exposure, namely, 3 hours 30 minutes. This exposure was sufficient to bring out some of the nebulosity in the Pleiades cluster.

The following data for the more prominent stars (all numbered in the figure) lying near the meteor track will serve to fix the actual path of the meteor.

No.	Star H.D.		R.A. (1900)	Dec. (1900)	Ptg. Mag.
			h m	° ′	
1	24534	χ Persei	3 49·1	+30 45	Var.
2	24167		45·8	30 52	6·3
3	25867	ψ Tauri	4 0·8	28 44	5·7
4	25823	41 Tauri	0·5	27 21	5·3
5	24970		3 53·0	26 55	7·4
6	25555	36 Tauri	58·4	23 50	6·1
7	24899		52·4	23 48	7·0
8	25201		55·1	22 57	6·5
9	24769		51·2	22 54	5·9
10	24740	32 Tauri	50·9	22 12	6·0
11	25604	37 Tauri	58·8	21 49	5·5
12	25680	39 Tauri	59·4	21 44	6·7

Fig. 2 shows the meteor as photographed with the triplet of focal length 70 cm. In this case the stars trailed a little, as is shown in the illustration. The exposure in this case was 5 hours.

The northern end of the trail (top of illustration) passes between the stars ξ and o Persei (not shown in fig. 1). The southern end lies between 37 and 32 Tauri, being approximately one-third of the distance between these two stars from 37 Tauri. The trail actually passed over the faint star H.D. 24167 near the beginning, and very close to H.D. 25201 near the end.

Tracing the path back on a celestial globe, the radiant point would lie on a line passing through Perseus, near the boundary between Cassiopeia and Camelopardalus, and through Cepheus and Draco.

During the exposures of the cameras in the Mond instrument the writer was taking stellar spectrum photographs with the 9-inch prismatic camera; thus he did not observe the meteor that was recorded with the former instrument. In order, however, to find out whether any visual records of this meteor, which must have been a very bright one, had been made by other observers elsewhere, he communicated with Mr. J. P. M. Prentice, the director of the Meteor Section of the British Astronomical Association. No information, however, has been received from him as to its having been seen by any person, owing most probably to the very late hour of the night at which it made its appearance.

It is interesting to note that the nucleus of the meteor is quite distinctly visible throughout that portion of the trail where the greatest volatilisation took place. This indicates that the hot vapours given off by the nucleus were not rendered sufficiently luminous to make their trail as intense as that of the glowing nucleus itself.

In the case of the meteor which I photographed, described and illustrated in 1922,* the emitted vapours were so bright that the nucleus could hardly be traced at all along the trail where the volatilisation was greatest. The probable explanation of this is that the 1922 meteor was very much brighter and possibly moved more slowly than that of the present year, thus tending to make the recorded trail of the nucleus and surrounding vapours of the same photographic intensity.

* *M.N.*, **83**, 92, 1922.

Fig 1.

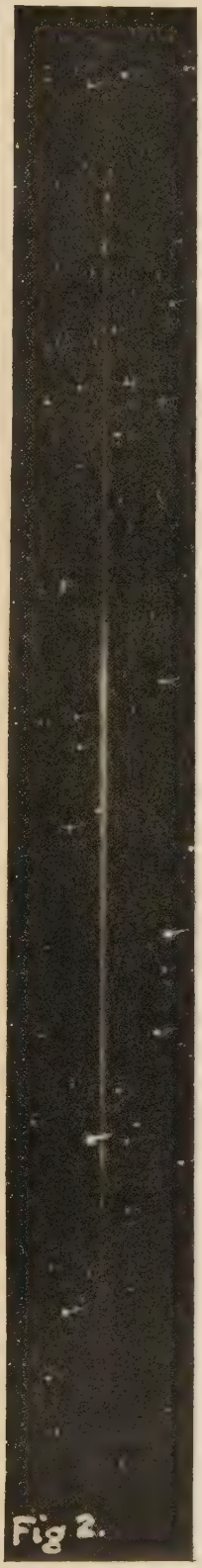


Fig 2.

Sometimes the volatilisation of a meteor is not homogeneous along its path, but exhibits bursts of brilliancy like that of the 1922 meteor. In the present instance this was not the case, as the meteor seems to have very rapidly reached its maximum brightness and then, much more slowly and homogeneously, become fainter until its trail disappeared altogether.

SUPPLEMENTARY NOTE ON THE ORBIT OF COMET PONS-COGGIA-WINNECKE-FORBES AT VARIOUS APPARITIONS.

A. C. D. Crommelin.

This note is supplementary to those in *M.N.*, **89**, 556, 1929 ; **90**, 232, 1929; and **90**, 322, 1930. In the first of these I referred to the possibility that the comet might be identical with that observed by Toscanelli in 1457 January ; a difficulty was presented by the fact that Celoria's elements for that apparition indicated a much smaller value of the inclination than that deduced from the recent returns. I have now deduced a new orbit from Toscanelli's positions, which gives a satisfactory agreement of the inclination with the modern value, though ω and Ω do not agree quite so well as before ; but they do not differ more than we might expect from the perturbations in 13 revolutions. My orbit and Celoria's are as follows (T is in G.M.T., and the equinox is that of 1457·0):

	First Orbit	Second Orbit
T	1457 Jan. 24·47	1457 Jan. 17·9794
ω	204° 18′	194° 54′·2
Ω	235° 4′	249° 39′·3
i	23° 45′	13° 15′·7
log q	9·8619	9·84718
Period	28 years (assumed)	Parabola
Computer	Crommelin	Celoria

A comparison of the observed positions with those deduced from the two orbits follows. Parallax and light-time have not been considered ; their effects would be trifling, and nearly the same on the different nights. The time of observation is given by Celoria as 6^h Paris M.T. on each evening.

Day	Longitude			Latitude		
	Observed	Computed		Observed	Computed	
		First Orbit	Second Orbit		First Orbit	Second Orbit
1457	° ′	° ′	° ′	° ′	° ′	° ′
Jan. 23	358 13·5	358 13·5	358 15·1	− 7 9·0	− 7 9·0	− 7 8·9
24	359 33·0	359 18·2	359 16·3	− 7 33·0	− 7 57·1	− 7 56·0
25	0 26·1	0 23·4	0 20·1	− 8 46·3	− 8 45·9	− 8 44·7
26	1 36·0	1 28·2	1 23·5	− 9 39·9	− 9 35·8	− 9 36·3
27	2 30·0	2 30·0	2 30·6	−10 29·7	−10 29·7	−10 25·5

My orbit represents the observations a little better than Celoria's, but I lay no stress on this, as the observations are liable to errors of several minutes. Toscanelli did not give measures, but plotted the comet's place each night on a star chart, from which Celoria deduced the longitude and latitude.

As I noted in my first paper, one-thirteenth of the interval from 1457 to 1818 is 27·77 years, which is very close to the present period of the comet ; this, combined with the close resemblance of the elements, makes the assumption of identity extremely probable. The comet must have declined in brightness ; in 1457 it was visible to the naked eye, and had a tail half a degree long, shown on Toscanelli's sketches. At the modern apparitions it was telescopic, and had no tail ; this fading is not surprising, many other comets having shown it.

It is of interest to quote the reference to this comet in Williams's *Chinese Comets*, p. 78 (I omit the Chinese dating) : "On 1457 January 14 another comet was seen in Stellar Division Peih ; it was half a cubit in length ; it went to the south-east ; it gradually lengthened until January 23, when it disappeared." The agreement in date, length of tail, and direction of motion leaves no doubt as to the identity of the comet with Toscanelli's. But Williams seems to have made a mistake as to Stellar Division Peih, which he here identifies with the longitude of Aldebaran and the Hyades ; his table on p. xxvii indicates another Peih, corresponding to the longitude of Gamma Pegasi ; comparison with Toscanelli shows that this is the one to select here. Each stellar division was a long strip running north and south, so that mention of a stellar division does not define the latitude of an object.

Reverting to the modern observations, I noted in *M.N.*, **90**, 324, that it was desirable to recompute the perturbations from 1818 to 1928, using shorter time-intervals. I have now carried out this suggestion, using 3° intervals of M instead of 9° as heretofore. The changes are inconsiderable except in relation to the values of the period of revolution at the different apparitions.

From the two revolutions 1818 to 1873, I find for the period in 1818, $27^{y}\cdot72189$; and in 1873, $28^{y}\cdot02702$. From the two revolutions 1873 to 1928, I find for the period in 1873, $28^{y}\cdot03554$; and in 1928, $27^{y}\cdot91266$. The discordance between the two determinations of the 1873 period was formerly $0^{y}\cdot0357$ or 13 days (*M.N.*, **89**, 561) ; it is now reduced to $0^{y}\cdot00852$ or 3·11 days. I rest content with that figure ; it could, doubtless, be further reduced by applying Cowell's method of mechanical quadratures to the four revolutions.

The calculated date of the next perihelion passage, which was given as 1956 October 3·2 in *M.N.*, **90**, 324, now becomes October 8. The change makes the conditions of observation somewhat more favourable ; they are best when perihelion falls early in December.

I have found the following as the dates of perihelion at the intermediate returns : 1845 September 29·8 and 1901 May 7·9.

A MODIFIED COMPARISON-IMAGE MICROMETER.

F. J. Hargreaves.

In *M.N.*, **92**, 1, 1931, there is described, under the title "A Comparison-Image Micrometer," an instrument in which artificial star images are projected into the focal plane of a telescope by means of a Swan cube. It was pointed out that serious loss of light occurred owing to the use of this cube, and a modification was mentioned in which this disadvantage could be overcome in part by providing a choice as to whether most light was to be lost in the real images or in the comparison images. It has now been found that a further modification is possible in which no loss of light whatever occurs.

In this further modification the two converging beams, one from the telescope object-glass or mirror and the other from the small projecting lens, are placed side by side like two cones in rolling contact, and the eyepiece is placed with its axis coincident with the line of contact, as in fig. 1, where *a* is the beam from the telescope, *b* the beam from the projecting lens, and *c* the eyepiece. Thus the eyepiece axis is slightly inclined to the axes of both beams, the two angles of inclination being equal and their value depending on the focal ratio of the telescope.

A well-corrected eyepiece can be tilted considerably without causing any visible alteration of the image at the centre of the field, and in fact even a tilt of 1 in 12 is not too great in the case of an "orthoscopic" eyepiece. This is the tilt necessary with the reflector with which the instrument is used, the focal ratio of which is $\frac{1}{6}$. Most telescopes used for measuring double stars have a much smaller focal ratio than this, so that it can confidently be expected that in the general case no adverse effect on the images themselves will be caused.

Owing to the fact that the two beams are symmetrically disposed with respect to the eyepiece, any geometrical effects which the tilt may have on the measurements are eliminated. Thus, when the images of two stars are in the plane containing the three optical axes, their apparent separation is less than it would be if the eyepiece were square to the focal plane. Exactly the same effect, however, is produced on the comparison images, and therefore no error is introduced into the determination of separation. The slight change in the apparent position-angle which occurs at what may be termed "inter-cardinal" points will also be eliminated for the same reason.

Two star images will not, in theory, be in focus together unless they lie in a plane perpendicular to the common plane of the three optical axes, but this difference of focus is vanishingly small. Neglecting the possibility of ocular accommodation, the malfocus, defined as the apparent angular diameter of the circle of confusion, varies directly as the square of the focal ratio and directly as the magnification. This malfocus cannot be detected

Fig. 1.

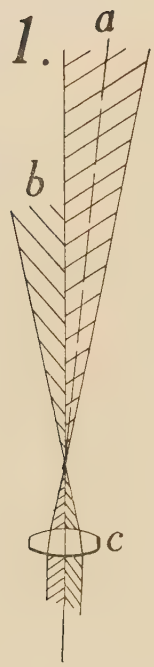


Fig. 2.

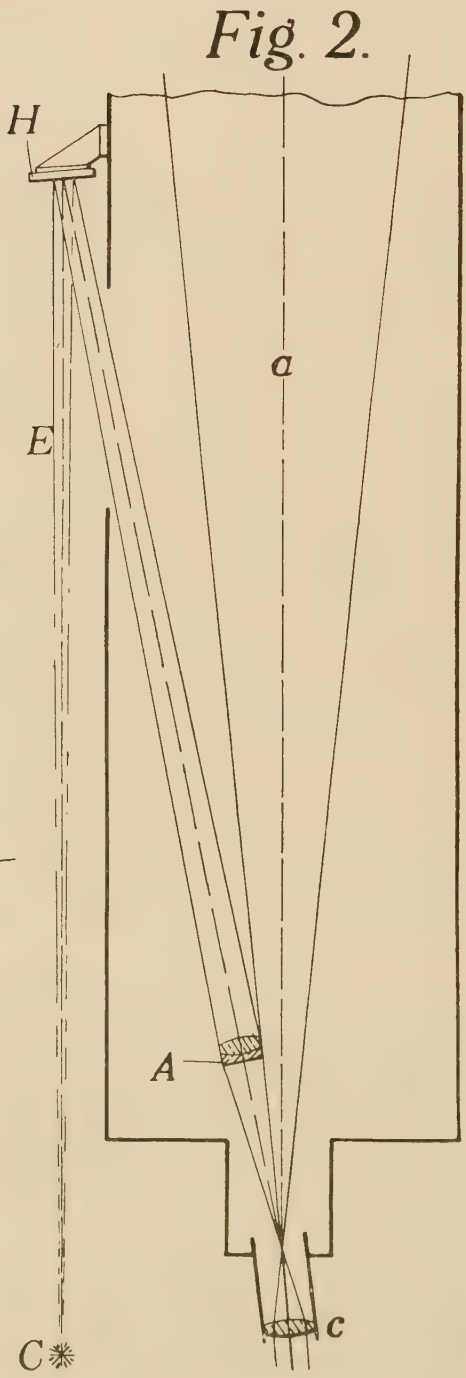


Fig. 3.

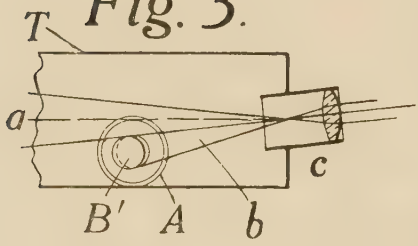
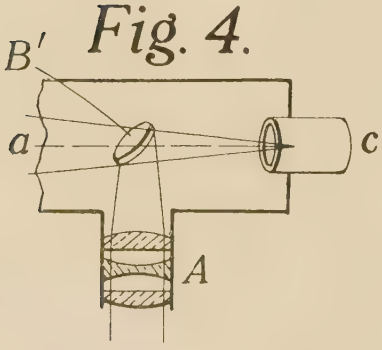


Fig. 4.



at all, either in the artificial images or in the actual star images, when the separation is 12″ and the magnification is 660 diameters, at a focal ratio of $\frac{1}{6}$; and as most telescopes used for double-star measurements have a much smaller focal ratio than this, and as a magnification of 660 diameters is more appropriate to separations below 1″, the question of malfocus can be disregarded.

The present modification makes it easier to apply the device to a refractor, because the axis of the comparison-image beam, being inclined to the axis of the object-glass, intersects the side of the telescope tube near its upper end, and light for this beam can therefore be introduced through a hole in the tube at this point, the projecting lens A being inside the tube near the focus. This arrangement is shown diagrammatically in fig. 2, in which the small mirror H is shown fixed near the hole E in the tube, to reflect the light from the artificial star C to the lens A. In large or moderately large refractors the device could be applied without disturbing the existing filar micrometer, which could therefore be used for calibration.

For a Newtonian reflector a small mirror B′ is placed in the eyepiece tube T (figs. 3 and 4) just clear of the beam from the speculum, and the projecting lens A is mounted excentrically so as to project its beam on to the mirror B′. This is the actual construction now in use.

In the modified form of the instrument the need for very bright artificial stars is not great, and the arrangement indicated in figs. 5 and 6 of the original paper is not necessary. Actually a small bulb with a straight filament has since been found, and this filament, used alone in place of the slit G, gives images of ample brightness. The use of the filament itself as a slit avoids a certain amount of difficulty in maintaining the light source D properly centred. In larger instruments, however, the necessarily longer filament might cause trouble owing to sagging.

PRELIMINARY VALUES OF THE VARIATION OF LATITUDE AT GREENWICH IN 1931.

(Communicated by the Astronomer Royal)

This paper contains the preliminary values of the latitude variation derived from observations made with the Cookson floating telescope.

The star places used were those derived from the final discussion of the observations for the period 1919–1926. The number of plates obtained was about the average, but owing to cloudy weather there was a comparatively small number of plates in April and December.

From 1931·127, each point has been obtained from approximately 20 consecutive plates grouped together: thus all points have approximately the same weight and each depends on a larger number of groups than formerly.

The measurement of the plates has been simplified by the purchase of a new measuring micrometer from Messrs. A. Hilger, Ltd. For purposes

of comparison with the old micrometer, the first point of the year was measured with both micrometers ; the results differed by 0″·001 only.

The following table gives the value of the variation (mean latitude *minus* observed latitude):—

Date	Variation	Number of Pairs
	″	
1931·044	+0·196	103
·127	+0·226	76
·217	+0·178	79
·321	+0·132	72
·413	+0·040	72
·491	−0·023	81
·590	−0·132	84
·671	−0·122	95
·747	−0·194	88
·804	+0·015	88
·844	−0·018	88
·933	+0·083	70
1932·036	+0·101	80

From a curve drawn to represent the observed results as closely as possible, the following corrections, applicable to observed north polar distances, were derived:—

	″		″
1931·0	+0·16	1931·6	−0·12
31·1	+0·22	31·7	−0·17
31·2	+0·20	31·8	−0·04
31·3	+0·14	31·9	+0·05
31·4	+0·06	1932·0	+0·10
31·5	−0·03		

Royal Observatory, Greenwich :
1932 *February.*

OCCULTATIONS OF STARS BY THE MOON OBSERVED AT BIRMINGHAM UNIVERSITY DURING THE YEAR 1931.

James Young.

Star	Mag.	Phase	G.M.T.	Corr.	χ	$\rho-\chi$	$\sigma'-\sigma$
			d h m s	s	° ′	° ′	″
ω Cancri	6·1	D	Feb. 1 20 6 5·4	−0·3	54 28	+ 47 16	−1·6
B.D. +27° 1337	6·4	D	Mar. 27 22 19 35·8	−0·3	82 31	+ 14 25	−0·4
406 B Tauri	5·6	D	Apr. 22 19 44 32·9	−0·3	87 0	+ 0 14	−0·2
τ Arietis *	5·1	R	July 11 1 44 34·0	−0·3	252 41	+178 28	−0·2
104 B Tauri	5·5	R	Oct. 28 18 44 17·1	−0·7	241 40	−167 44	+2·7
98 B Piscium †	6·3	D	Dec. 17 18 52 29·1	−0·2	13 16	+ 48 17	−1·8
ε Piscium †	4·4	D	Dec. 18 21 24 17·4	−0·3	0 37	+ 61 55	−0·9
ε Piscium ‡	4·4	R	Dec. 18 22 7 48·9	−0·5	291 6	+131 29	+1·9

* Reappearance sensibly gradual. † Two observers.
‡ Bright limb reappearance.

The observations were all made with the 7-inch Cooke refractor. The G.M.Ts. were determined from Riefler No. 245, the rate of which never varies by more than $0^s.2$ per day, and the error of which is observed daily from the B.B.C. time-signals. The stop-watch method was used, the watch being stopped at a definite tick of the Riefler clock heard by telephone from the clock room.

The G.M.Ts. shown in the above table have been corrected for personal error and lag in starting the stop-watch. The correction for a normal occultation is $-0^s.3$, but this value is sometimes adjusted by the observer according to his impression at the time. The actual total corrections applied in individual cases are shown in the fifth column of the above table.

The observations were reduced independently by Mr. J. D. McNeile and the observer. The usual correction of $+0^h.0030$ was made to the G.M.Ts. as given above, to allow for the mean longitude of the Moon being $6''.0$ ahead of its tabular value, as suggested by Professor E. W. Brown.

The co-ordinates of the observatory are given in the *Nautical Almanac* for 1932.

Erratum in Professor Brown's paper "On a Method of solving Kepler's Equation": p. 104, in the line after equation (1), *for* $e^5/46$ or $e^6/120$ *read* $e^6/46$ or $e^5/120$.